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Hierarchical Segmentation of Polarimetric SAR Images

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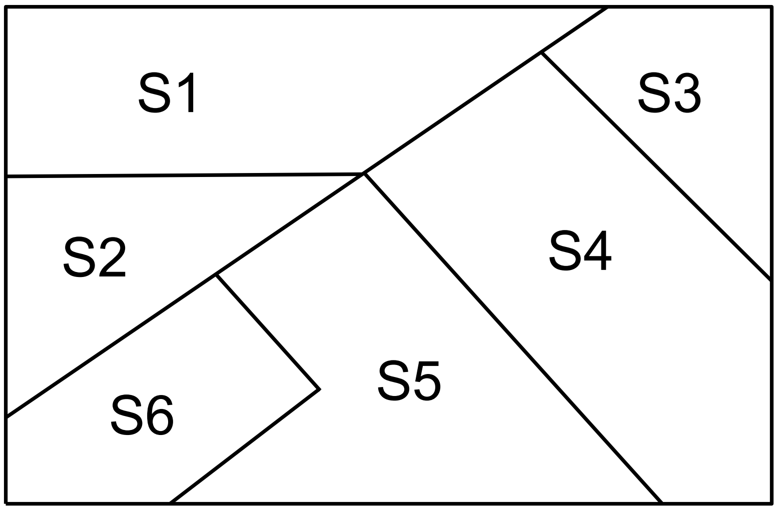
Hierarchical Segmentation of Polarimetric SAR Images

- Hierarchical Image Segmentation
- As a maximum likelihood estimation problem
- Segmentation of polarimetric images
- Segment sizes – shape constraints
- Results

Image Segmentation is the division of the image plane into regions



Two basic questions:

1- **What** kind of regions do we want ?

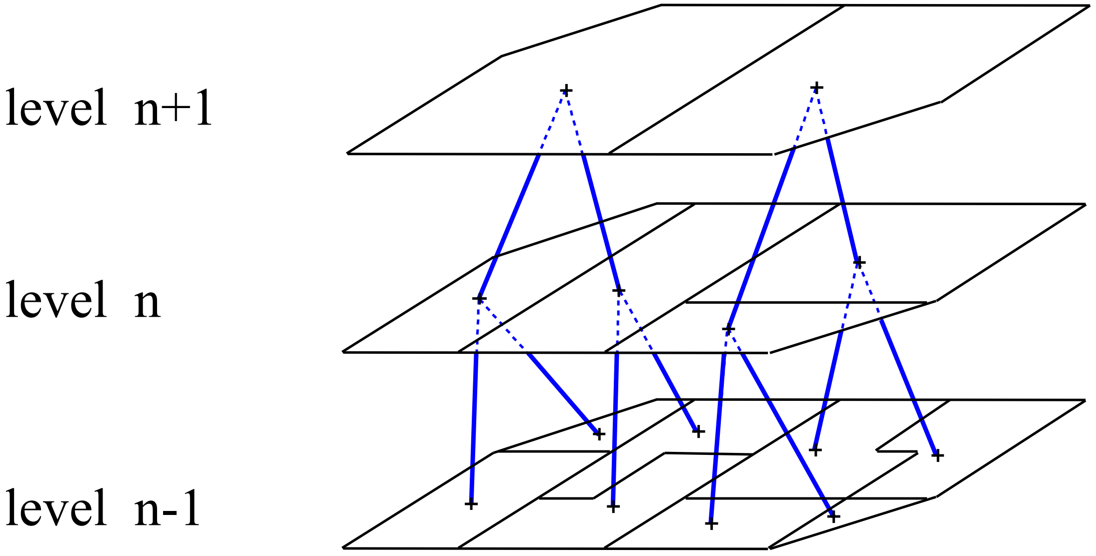
- Homogeneous regions
- Segment similarity

2- **How** can we obtain them ?

- Algorithm design

HIERARCHICAL SEGMENTATION BY STEP-WISE OPTIMISATION

A hierarchical segmentation begins with an initial partition P^0 (with N segments) and then sequentially merges these segments.



Segment tree

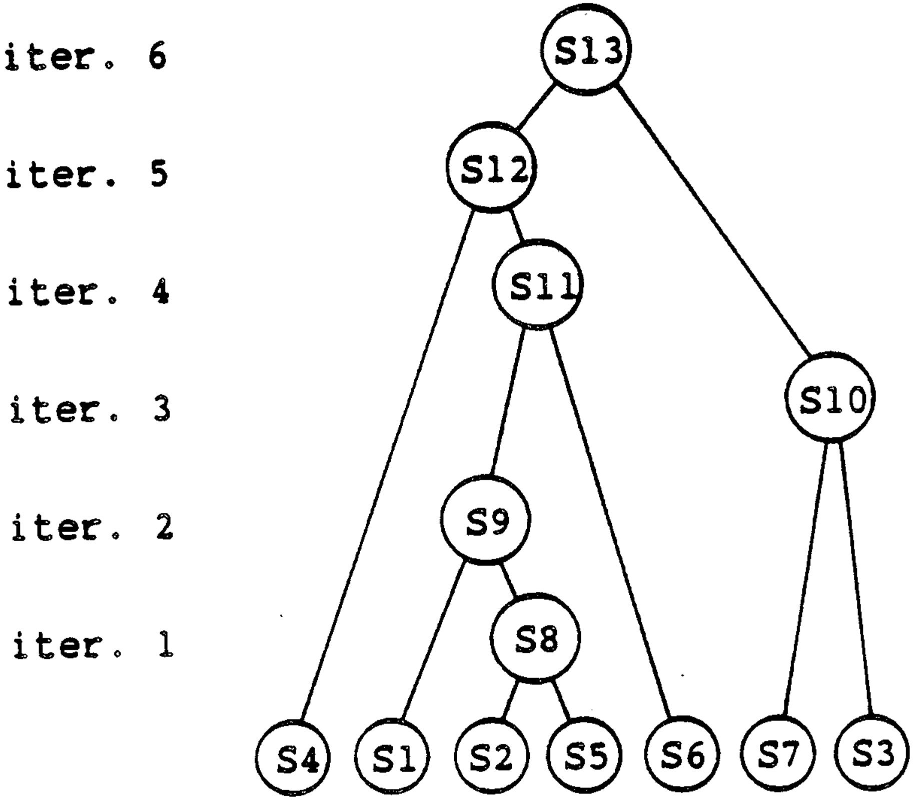
STEP-WISE OPTIMISATION

- A criterion, corresponding to a measure of segment similarity, is used to define which segments to merge.

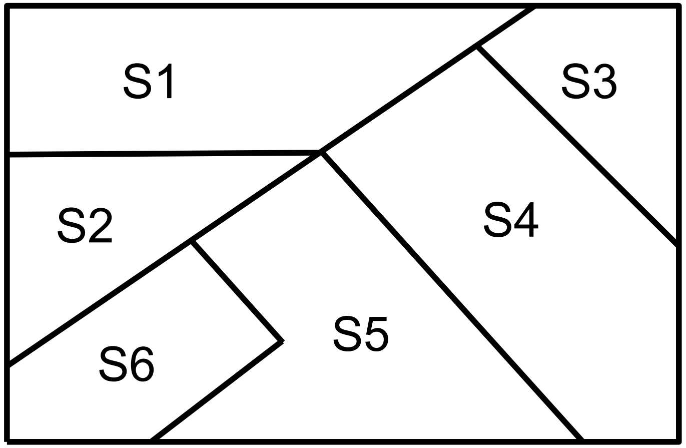
- At each iteration, an optimization process finds the two most similar segments and merges them.

- This can be represented by a segment tree, one node per iteration, where only the two most similar segments are merged.

Sequence of segment merges.



SEGMENTATION AS MAXIMUM LIKELIHOOD ESTIMATION



1) need a partition of the image

$$P = \{s_k\}, \quad s_k = \{i\} \subset I$$

2) need statistical parameters

$$\theta = \{\theta_s\}, \quad s \in P$$

3) need an image probability model

$$p(x_i \mid \theta_s)$$

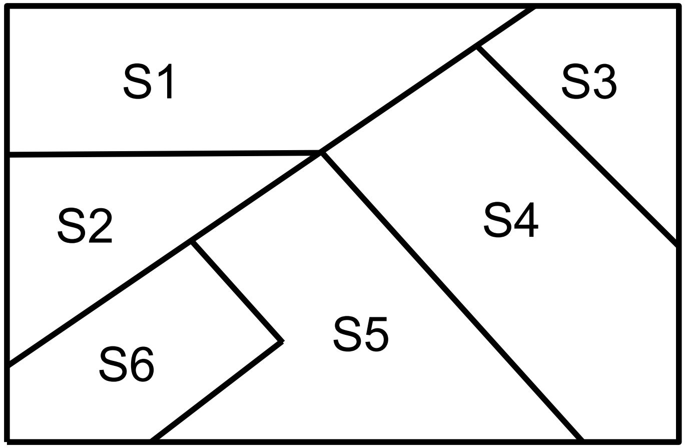
x_i are conditionally independent

Given an image $X = \{x_i\}, \quad i \in I$

the likelihood of $\theta = \{\theta_s\}, \quad P$

is $L(\theta, P \mid X) = p(X \mid \theta, P)$

$$L(\theta, P \mid X) = \prod_{i \in I} p(x_i \mid \theta_{s(i)}) \Big|_P$$

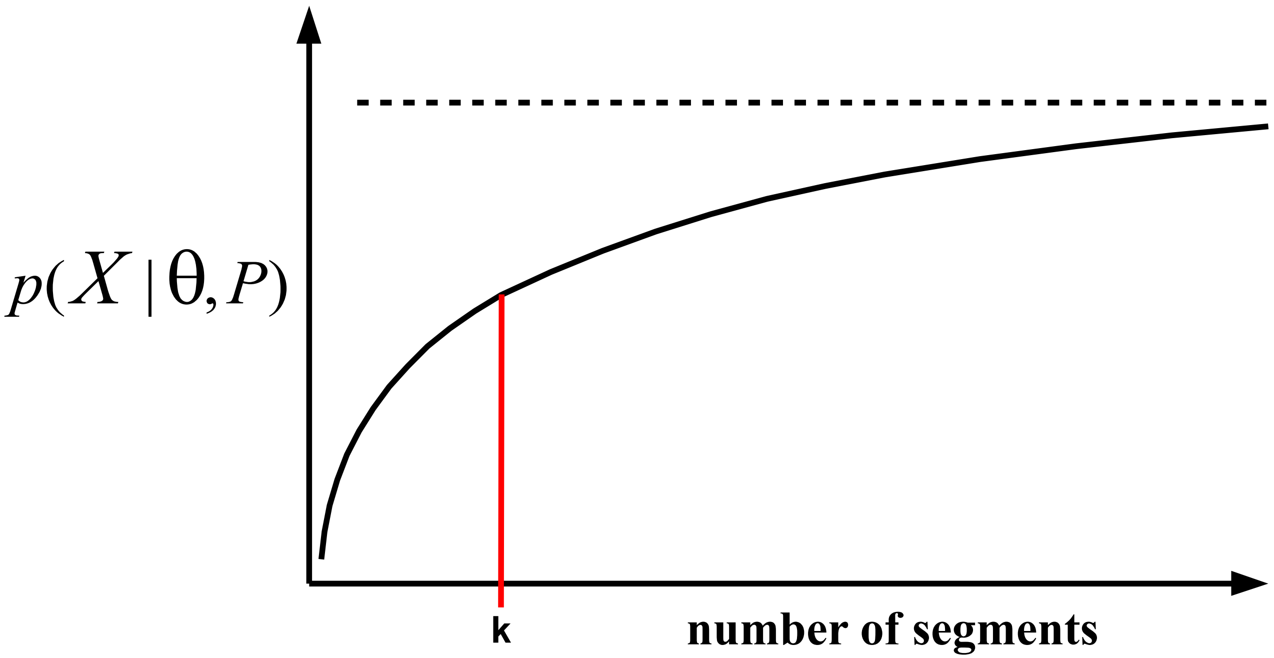


The segmentation problem is to find the partition that maximizes the likelihood.

Global search – too many possible partitions.

θ_s **is derived from statistics calculated over a segment *s*.**

The maximum likelihood increases with the number of segments

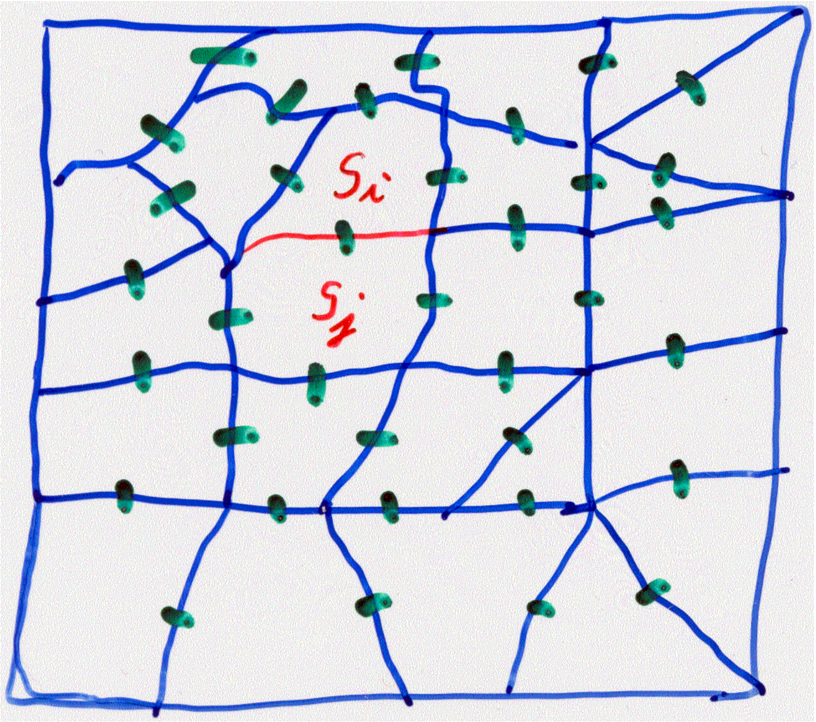


Can't find the optimum partition with k segments, P_k
Too many, except for P_1 and P_{nxn}.

Hierarchical segmentation
→ get P_k from P_{k+1} by merging 2 segments.

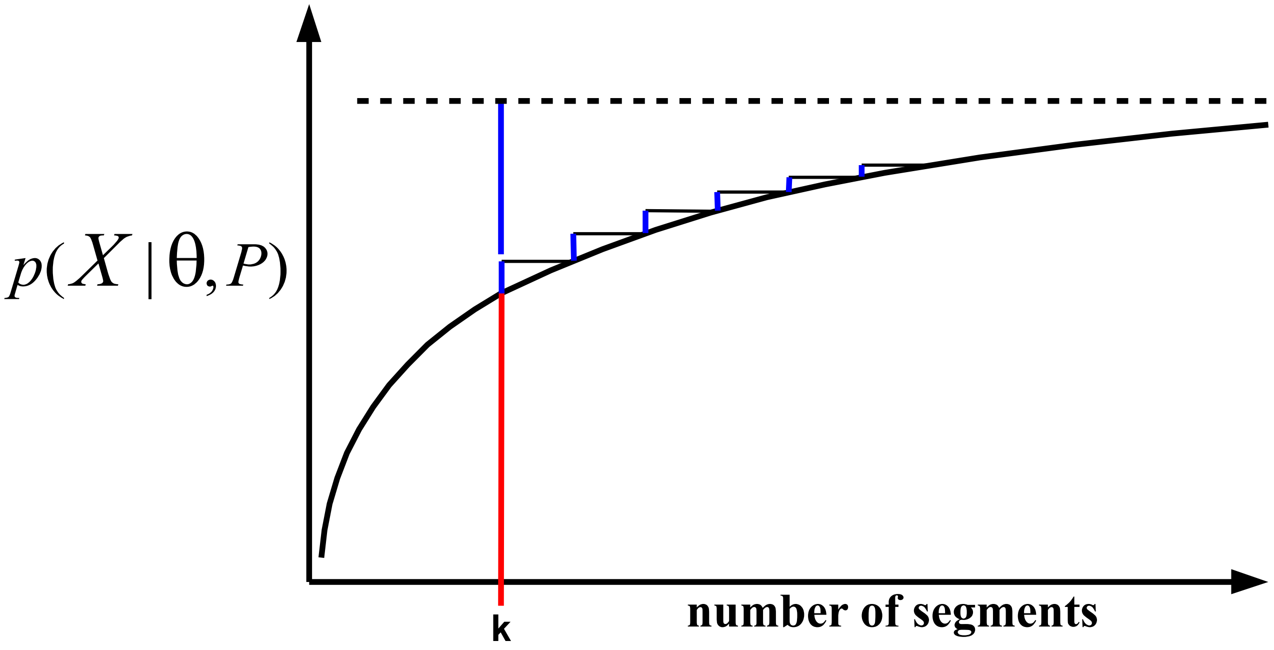
Stepwise optimization

- examine each adjacent segment pair
- merge the pair that minimizes the criterion



Merging criterion:

merge the 2 segments producing the smallest decrease of the maximum likelihood (stepwise optimization)



Sub-optimum within hierarchical merging framework.

Log likelihood form

$$\ln\left(L(\theta, P \mid X)\right) = \ln\left(\prod_{i\in I} p(x_i \mid \theta_{s(i)})\right) = \sum_{i\in I} \ln\left(p(x_i \mid \theta_{s(i)})\right)$$

Summation inside region

$$\sum_{s\in P} \sum_{i\in s} \ln\left(p(x_i \mid \theta_s)\right) = \sum_{s\in P} LML(s)$$

Criterion → cost of merging 2 segments

$$\Delta = LML(s_i) + LML(s_j) - LML(s_i \cup s_j)$$

$$\Delta = \sum_{x\in s_i} \ln\left(p(x \mid \theta_{s_i})\right) + \sum_{x\in s_j} \ln\left(p(x \mid \theta_{s_j})\right) - \sum_{x\in s_i\cup s_j} \ln\left(p(x \mid \theta_{s_i\cup s_j})\right)$$

minimize $|\Delta|$

POLARIMETRIC SAR IMAGE

Multi-channel image – 3 complex elements

$$x = \begin{bmatrix} hh \\ hv \\ vv \end{bmatrix}$$

each element has
a zero mean circular
gaussian distribution

Complex gaussian pdf (Σ is the covariance matrix)

$$p(x \mid \Sigma) = \frac{1}{\pi^3 |\Sigma|} \exp\left(-x^* \Sigma^{-1} x\right)$$

x^* is the complex conjugate transpose of x

The best maximum likelihood estimate of Σ is the covariance calculated over the region (segment)

$$\hat{\Sigma} = C = \frac{1}{n_s} \sum_{x \in s} x\, x^*$$

n_s is the number of pixels in segment s

$$\boldsymbol{C} = \frac{1}{n} \begin{bmatrix} \sum hh\, hh^* & \sum hh\, hv^* & \sum hh\, vv^* \\ \sum hv\, hh^* & \sum hv\, hv^* & \sum hv\, vv^* \\ \sum vv\, hh^* & \sum vv\, hv^* & \sum vv\, vv^* \end{bmatrix}$$

***LML* for a region *s* is**

$$LML(s) = \sum_{x \in s} \ln\left(p(x \mid C_s)\right) = \sum_{x \in s} \ln\left(\frac{1}{\pi^3 |C_s|} \exp\left(-x^* C_s^{-1} x\right)\right)$$

$$= \sum_{x \in s} \left[-\ln \pi^3 - \ln|C_s| - x^* C_s^{-1} x\right]$$

$$= -n_s \ln \pi^3 - n_s \ln|C_s| - \sum_{x \in s} x^* C_s^{-1} x$$

$$= -n_s \ln|C_s| - n_s \ln \pi^3 - 3n_s$$

constant term for the whole image

The variation produced by merging 2 segments is

$$\begin{aligned}\Delta &= LML(s_i) + LML(s_j) - LML(s_i \cup s_j) \\ &= -n_{si} \ln\left|C_{si}\right| - n_{sj} \ln\left|C_{sj}\right| + (n_{si} + n_{sj}) \ln\left|C_{si\cup sj}\right|\end{aligned}$$

Hierarchical segmentation:

at each iteration, merge the 2 segments that minimize the stepwise criterion $C_{\mathbf{i,j}}$

$$C_{i,j} = (n_{si} + n_{sj}) \ln\left|C_{si\cup sj}\right| - n_{si} \ln\left|C_{si}\right| - n_{sj} \ln\left|C_{sj}\right|$$

SEGMENTATION BY HYPOTHESIS TESTING

Test the similarity of segment covariances $C_i = C_j = C$

- merge segment with same covariance

Use the difference of determinant logarithms as a test statistic

$$C_{i,j} = K\left\{(n_{si}+n_{sj})\ln\left|C_{si\cup sj}\right| - n_{si}\ln\left|C_{si}\right| - n_{sj}\ln\left|C_{sj}\right|\right\}$$

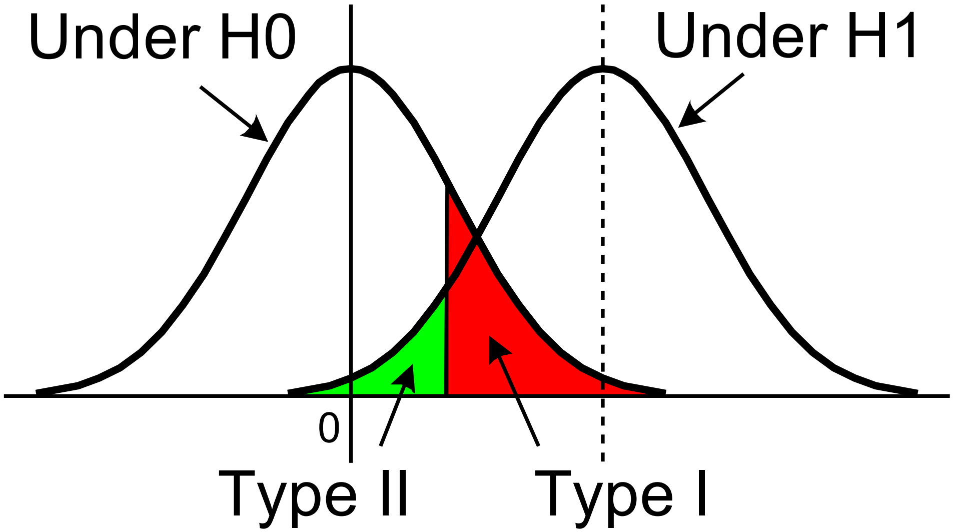
With the scaling factor *K*, the statistic is approximately distributed as a chi-squared variable with 6 degrees of freedom as n_{si} and n_{sj} become large.

$$K = 1 - {}^{13}\!/_{12}(1/n_{si} + 1/n_{sj} - 1/(n_{si}+n_{sj}))$$

Segmentation by hypothesis testing

Two hypothesis
H0: segments are similar
H1: segments are different

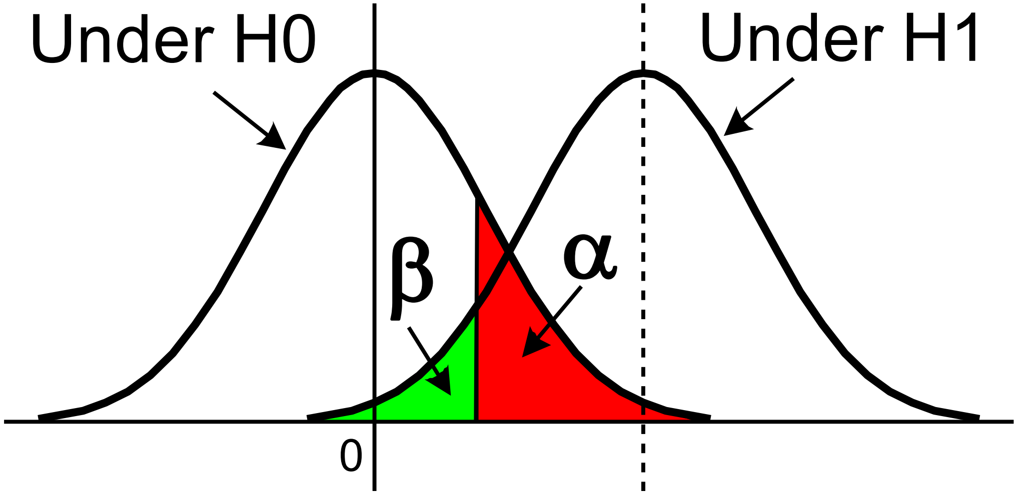
Distributions of the statistic d under H0 and H1



Two types of errors
Type I: not merging similar segments
Type II: merging different segments

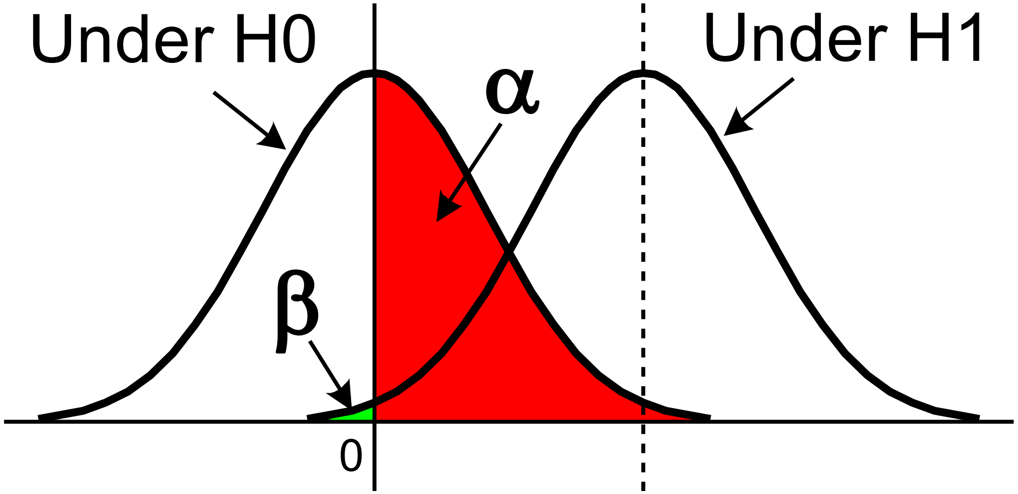
α = Prob(Type I errors)

β = Prob(Type II errors)



Select the threshold to minimise α or β, but not both simultaneously

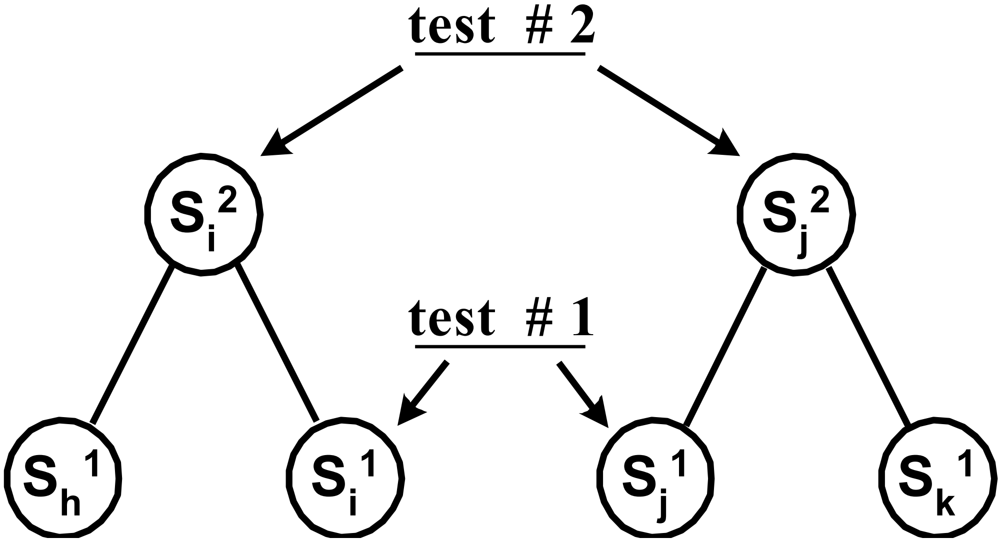
In hierarchical segmentation, type II errors (merging different segments) can not be corrected, while type I errors can be corrected later on.



The distribution of H1 and β are unknown.
Reduce β by increasing α.

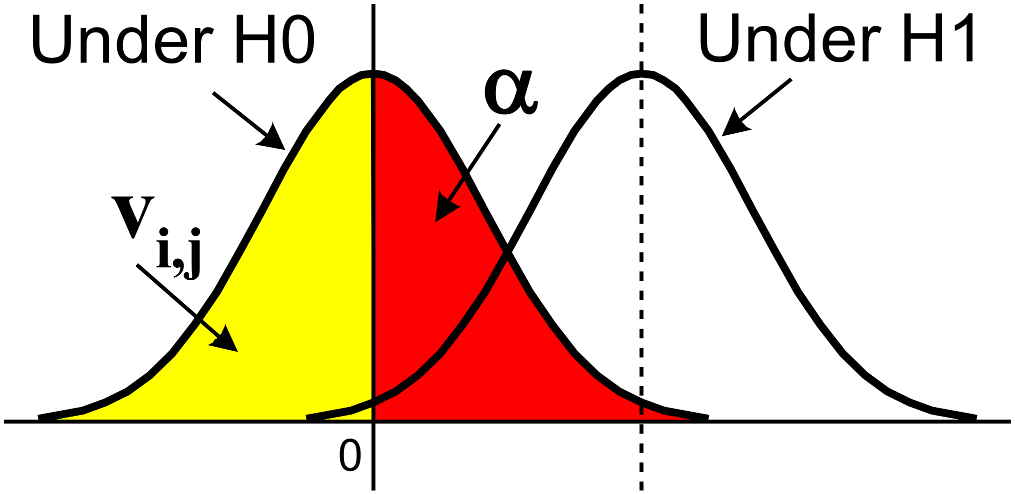
Sequential testing:
α will be reduced as segment sizes increase.

$\alpha_{1+2+\ldots} \leq$ **minimum**($\alpha_1, \alpha_2, \ldots$)
$\beta_{1+2+\ldots} \geq$ **maximum**($\beta_1, \beta_2, \ldots$)



Stepwise criterion

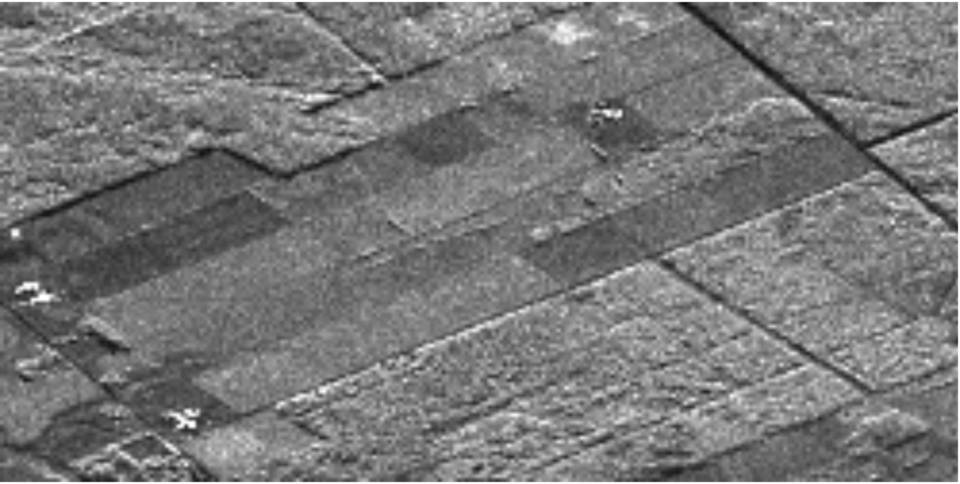
Find and merge the segment pair (i, j) that minimizes $V_{i,j}$ (= 1 - α).



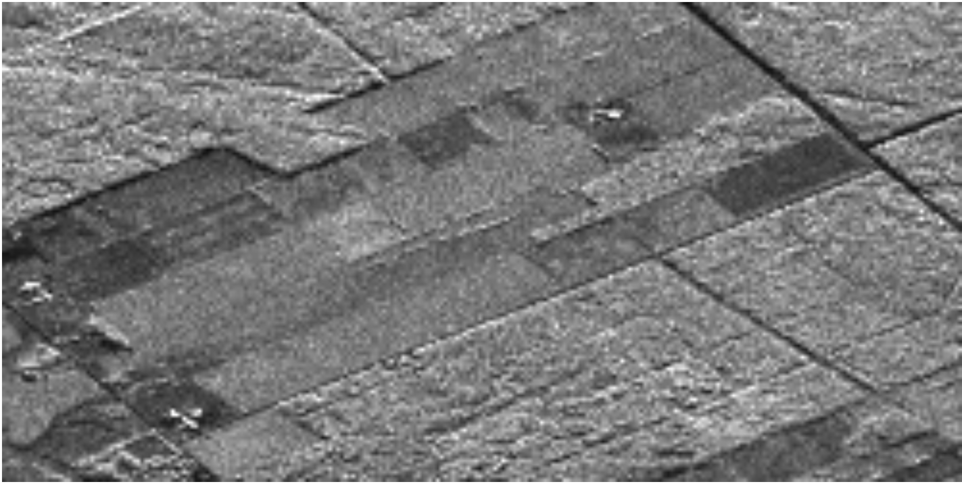
$$V_{i,j} = \mathrm{Prob}(\, d \le d_{i,j}\ ;\ H0\,) \quad (= 1 - \alpha\ ).$$

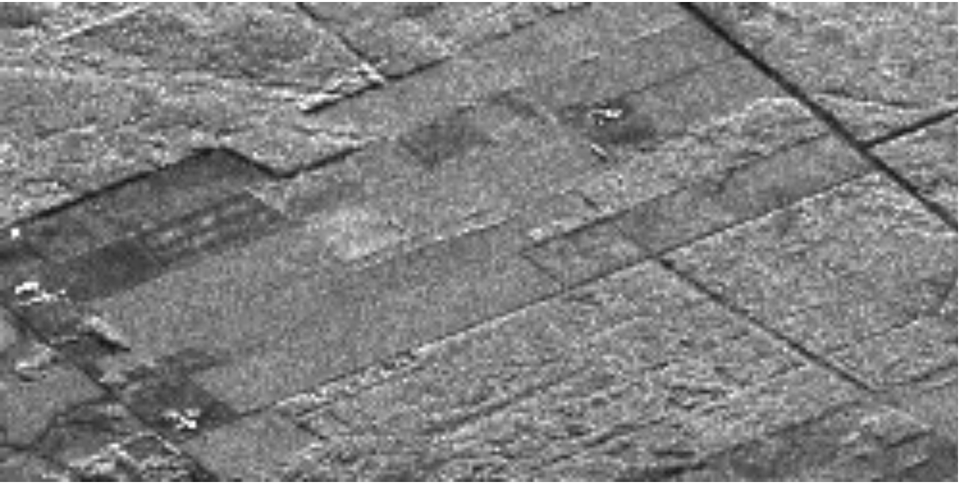
Amplitude values

|hh|

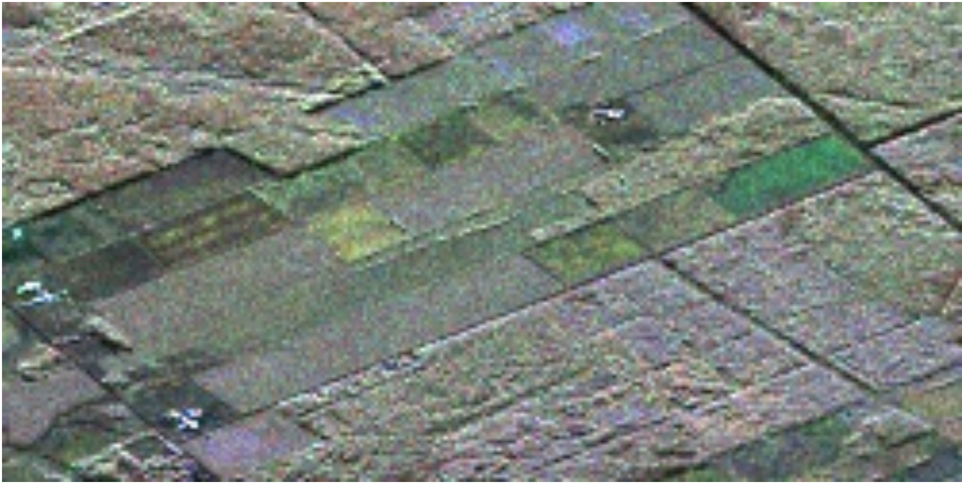


|hv|





|vv|

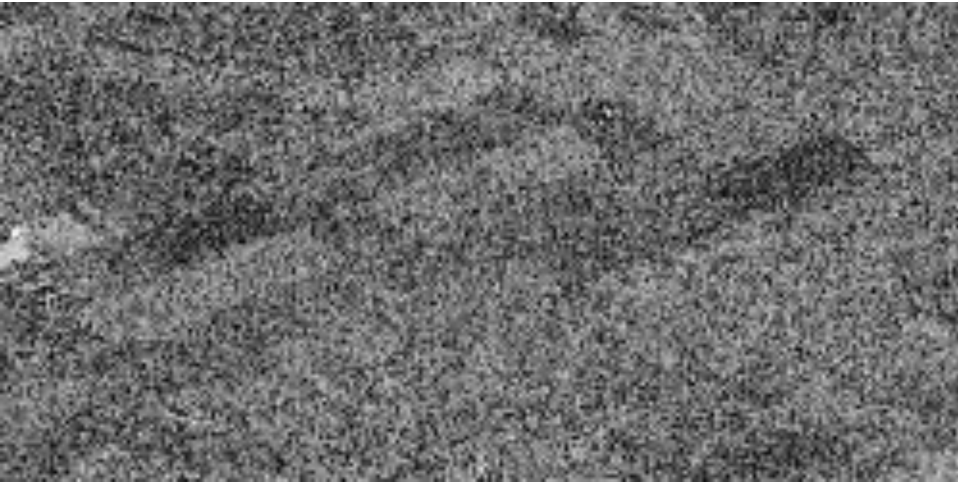


|hh| / |hv| / |vv|

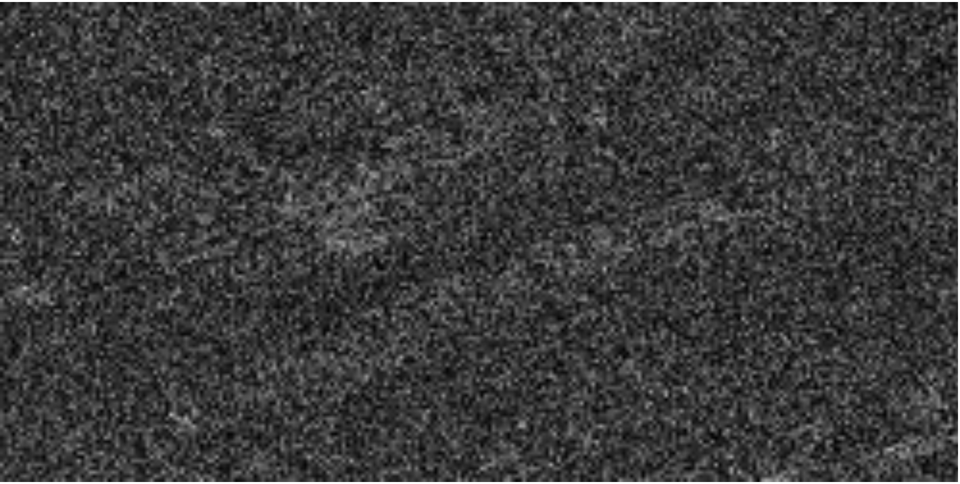
80 pixels / cell

Correlation – module   (0 – 1)

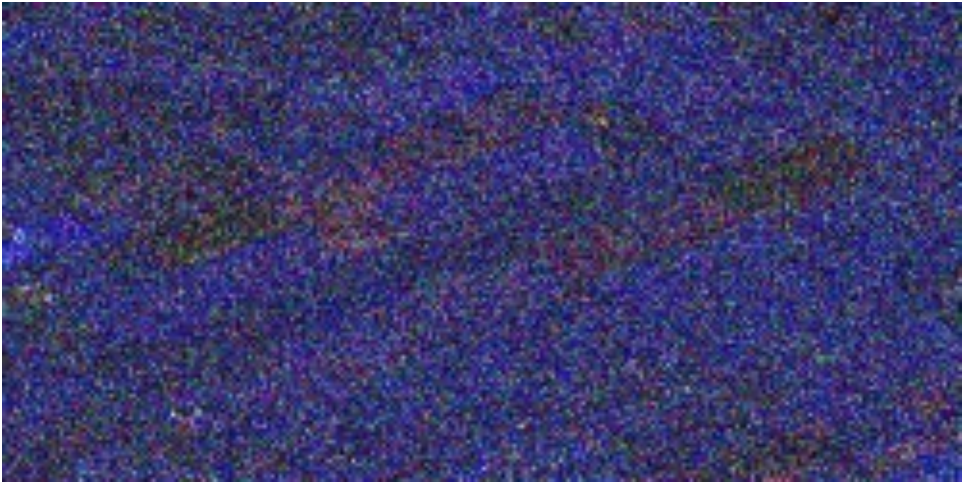
hh vv*



hh hv*



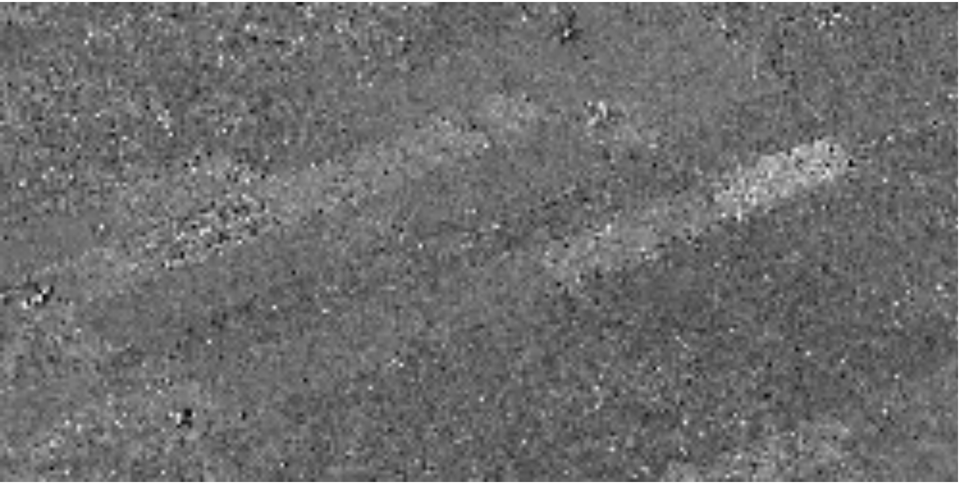
vv hv*



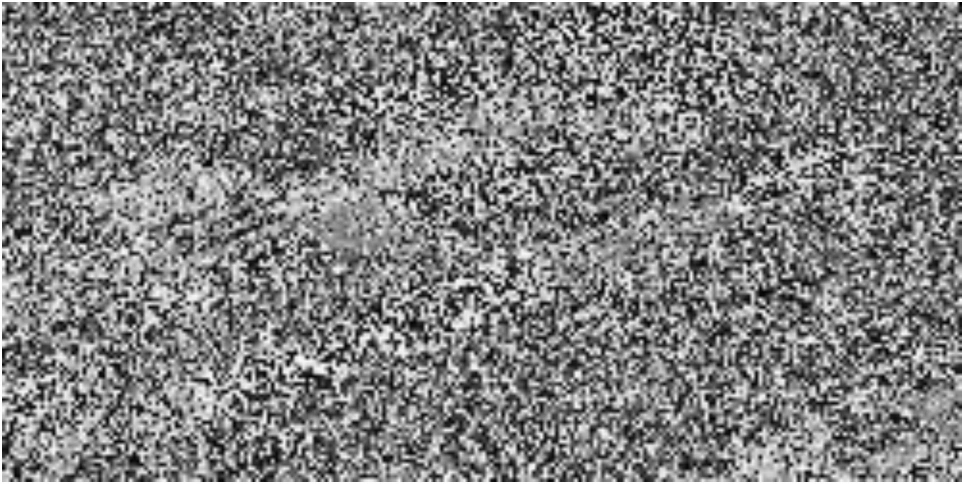
hh vv*/ hh hv*/ vv hv*

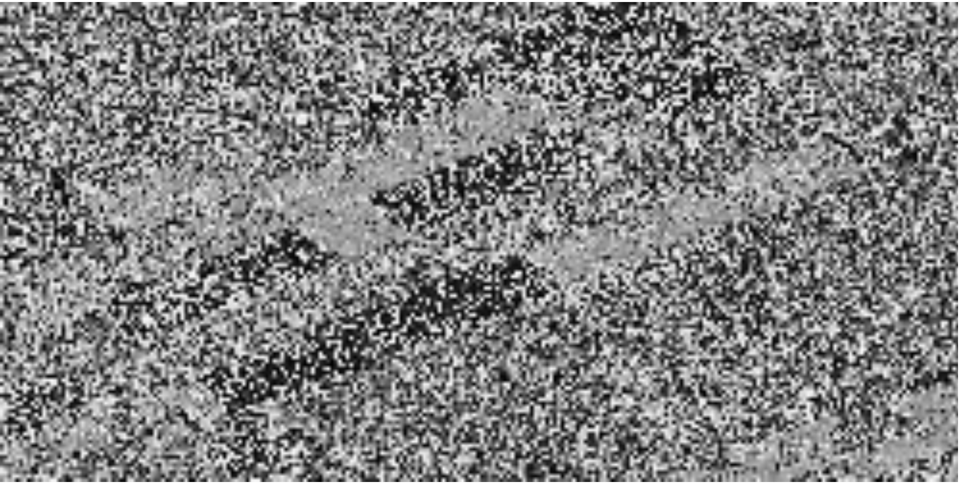
Correlation – phase (-180° – 180°)

hh vv*

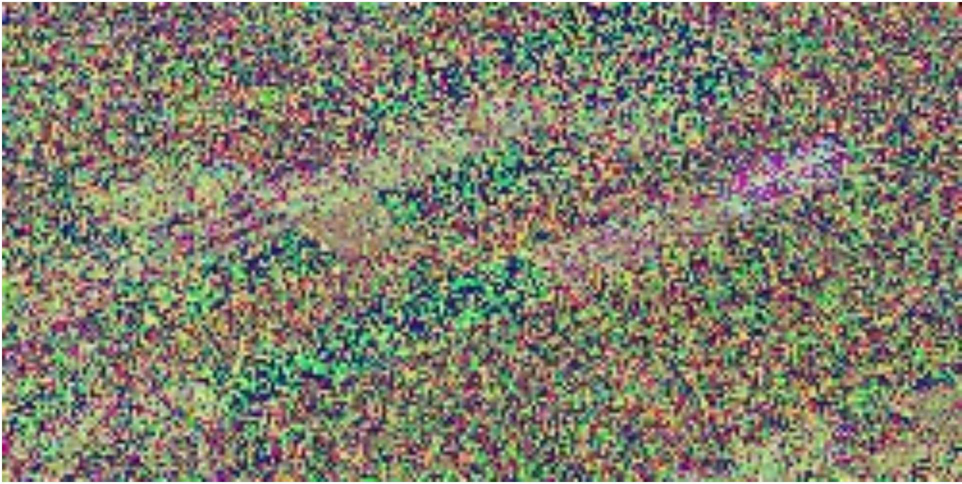


hh hv*





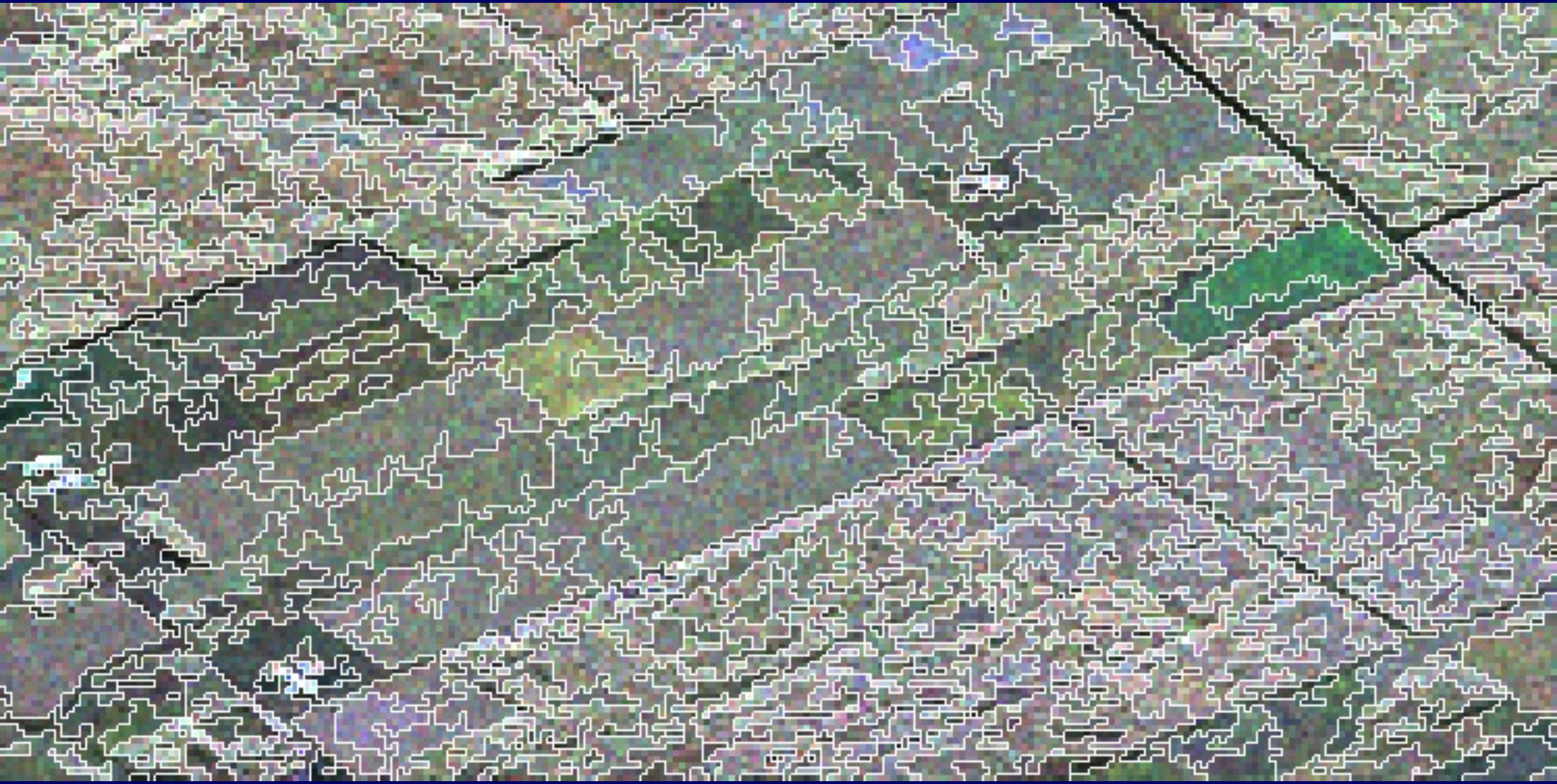
vv hv*



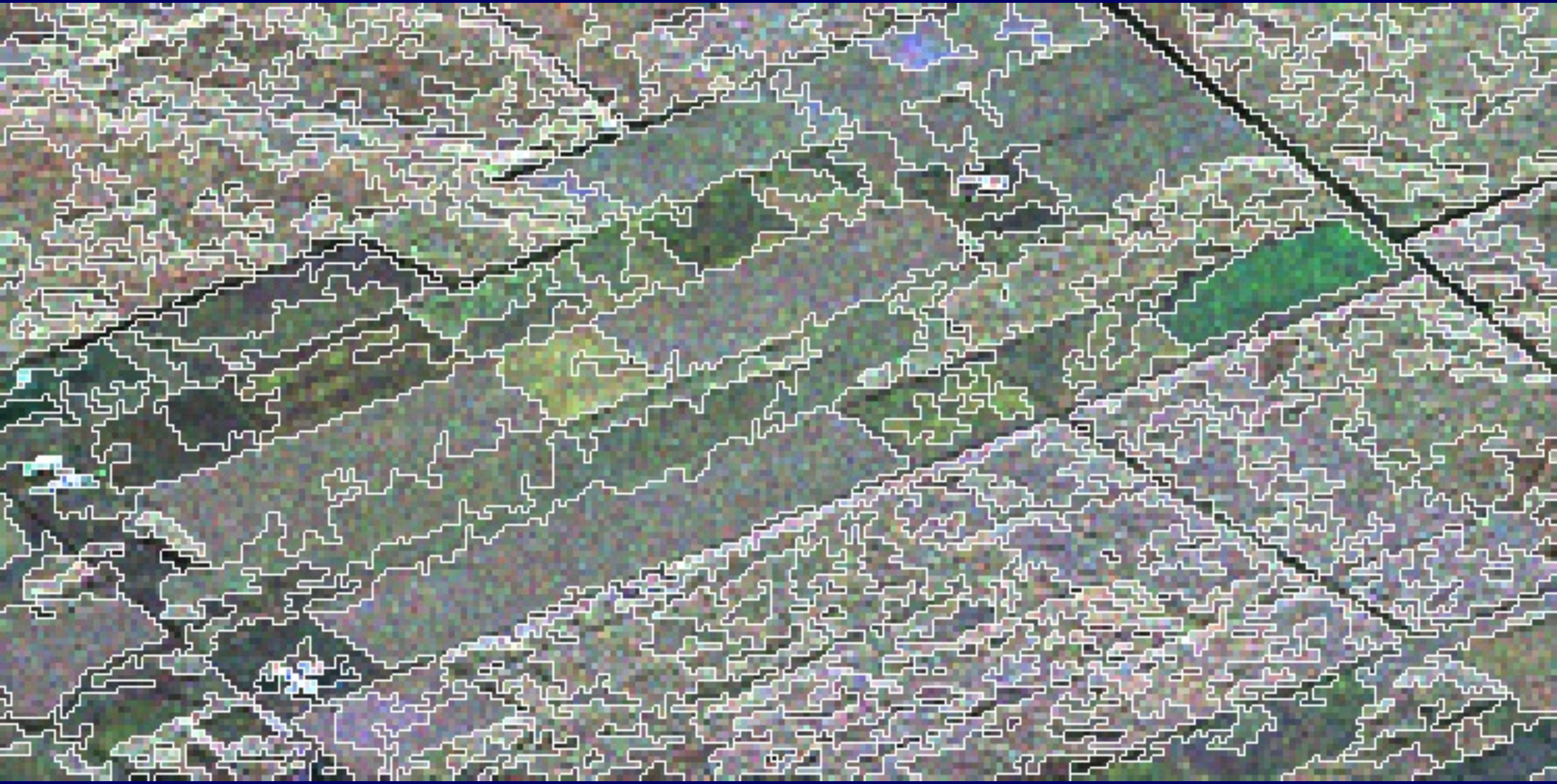
hh vv*/ hh hv*/ vv hv*

Amplitude image

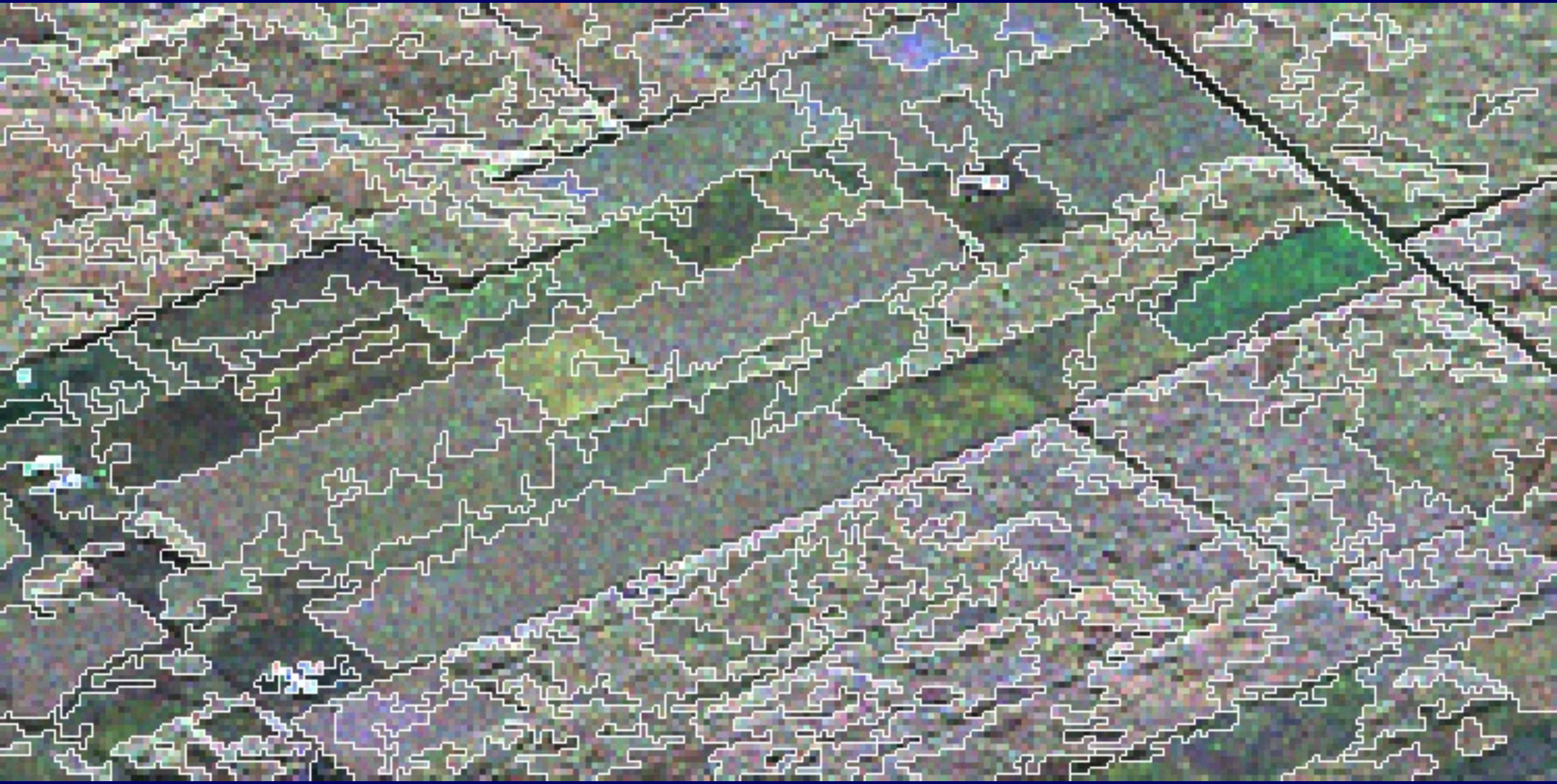
1000 segments



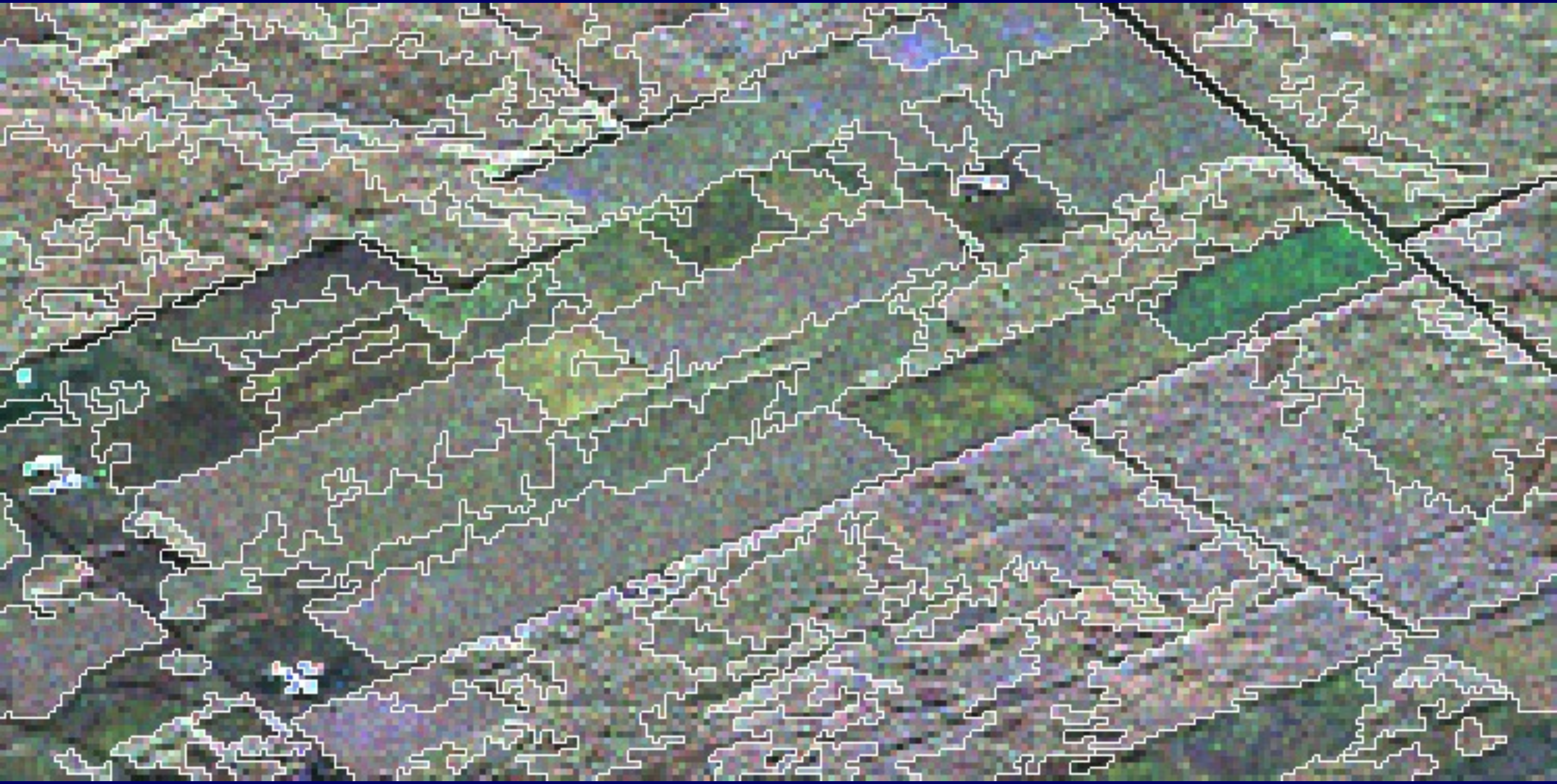
500 segments



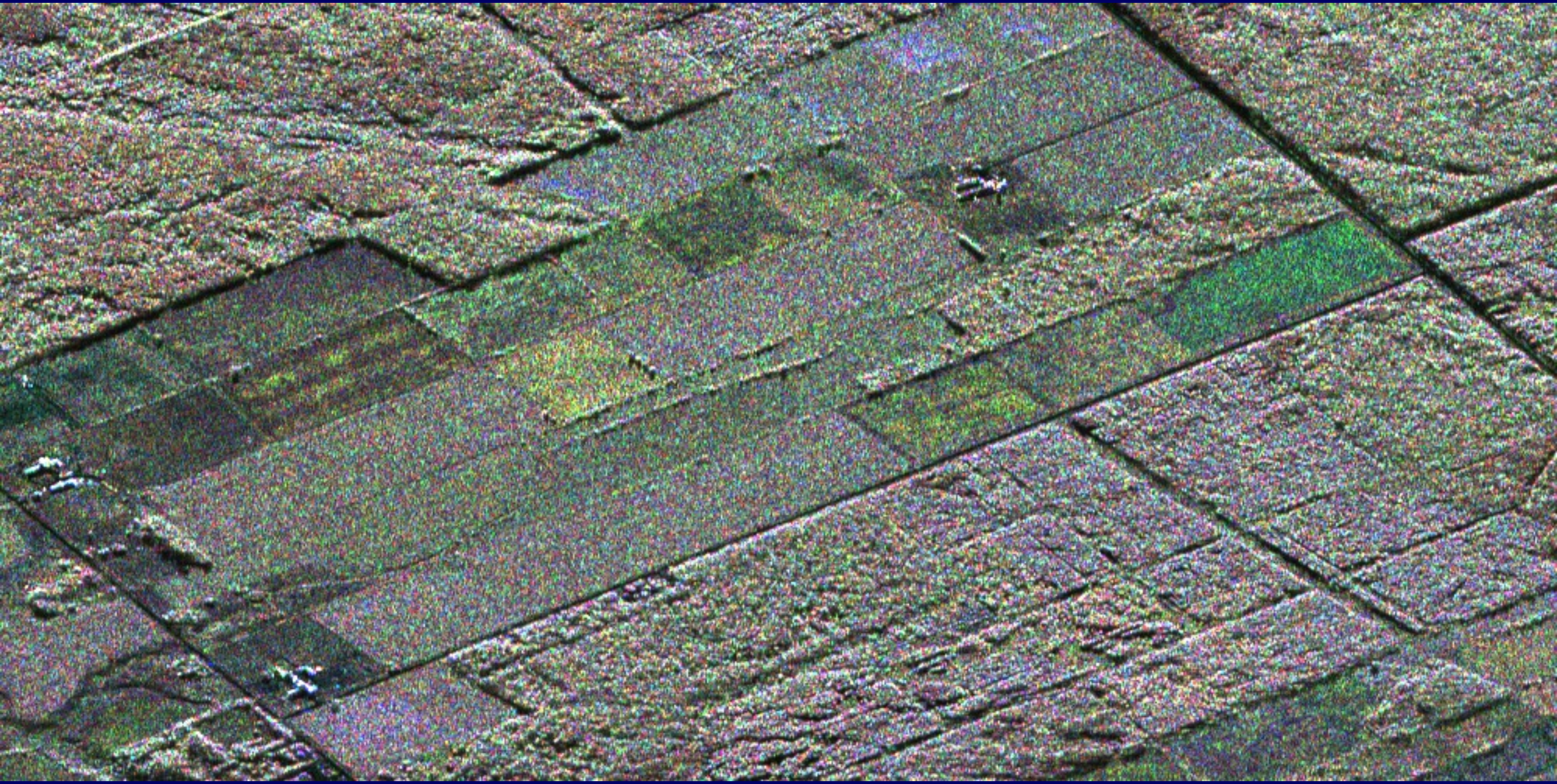
200 segments



100 segments



Amplitude image



5 pixels / cell

CRITERION FOR SMALL SEGMENTS

The determinant |C| is null for small segments

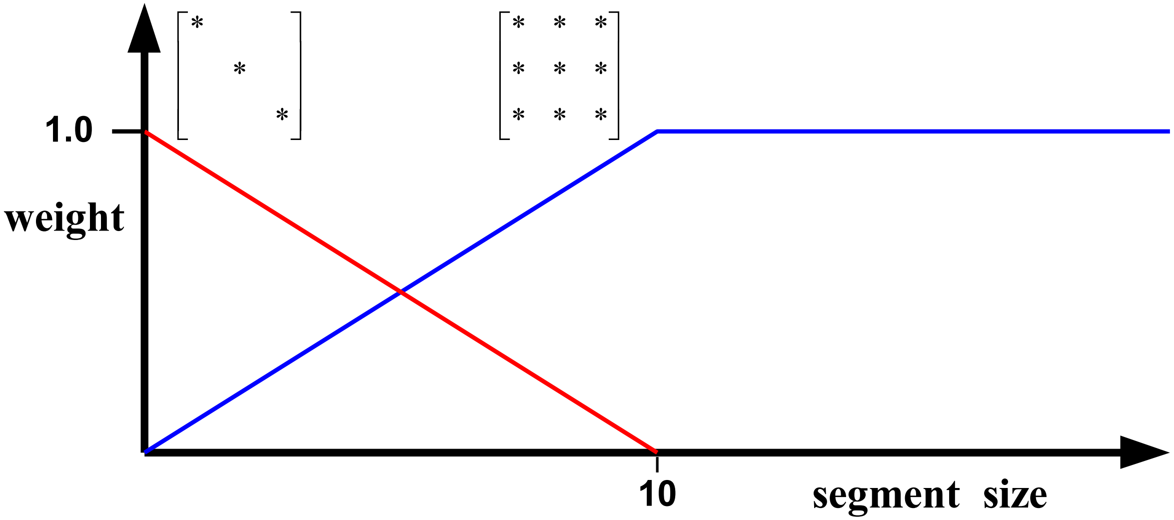
$$\boldsymbol{C} = \frac{1}{n}\begin{bmatrix} \sum hh\,hh^* & \sum hh\,hv^* & \sum hh\,vv^* \\ \sum hv\,hh^* & \sum hv\,hv^* & \sum hv\,vv^* \\ \sum vv\,hh^* & \sum vv\,hv^* & \sum vv\,vv^* \end{bmatrix}$$

Reduce covariance matrix model for small segments

$$\frac{1}{n}\begin{bmatrix} \sum hh\,hh^* & 0 & \sum hh\,vv^* \\ 0 & \sum hv\,hv^* & 0 \\ \sum vv\,hh^* & 0 & \sum vv\,vv^* \end{bmatrix}$$

$$\frac{1}{n}\begin{bmatrix} \sum hh\,hh^* & 0 & 0 \\ 0 & \sum hv\,hv^* & 0 \\ 0 & 0 & \sum vv\,vv^* \end{bmatrix}$$

Gradual transition between models



SEGMENT SHAPE CRITERIA

High speckle noise

→ first merges produce ill formed segments

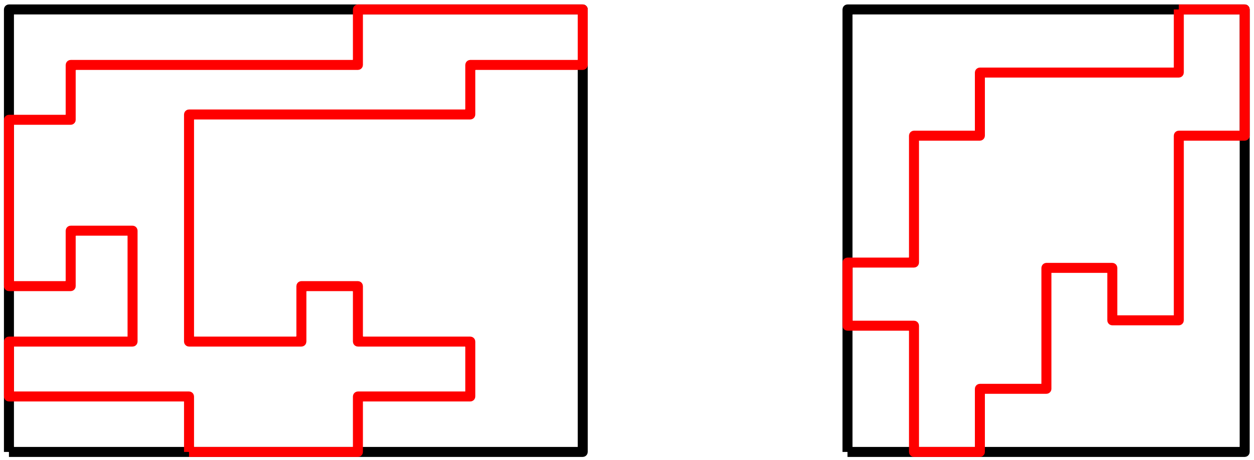
•Bonding box – perimeter	Cp
•Bonding box – area	Ca
•Contour length	Cl

New criteria

$$\mathrm{C}_{\mathrm{i,j}}^{contour} = \mathrm{C}_{\mathrm{i,j}}^{polar} \times Cp^{2} \times Ca \times Cl$$

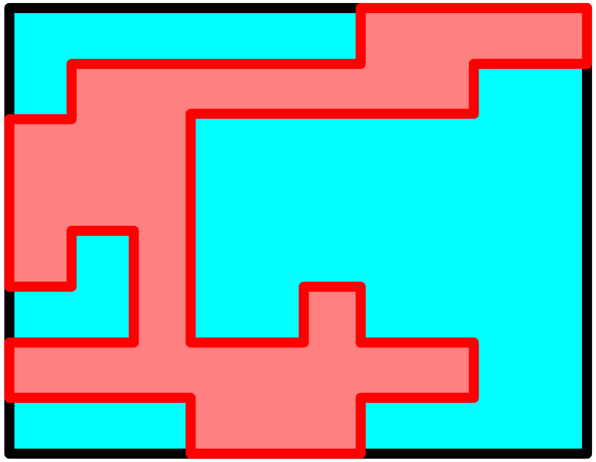
Bonding box – perimeter

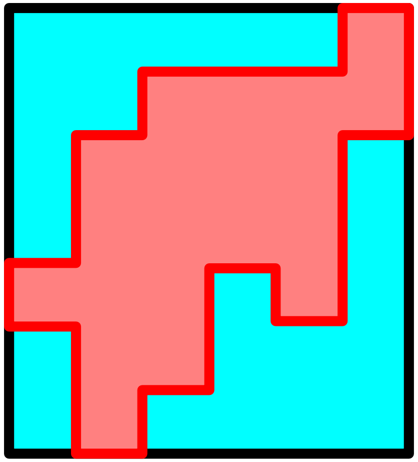
$$Cp = \frac{perimeter\ of\ S_i \cup S_j}{perimeter\ of\ bonding\ box}$$



Bonding box – area

$$Ca = \frac{area\ of\ bonding\ box}{area\ of\ S_i \cup S_j}$$



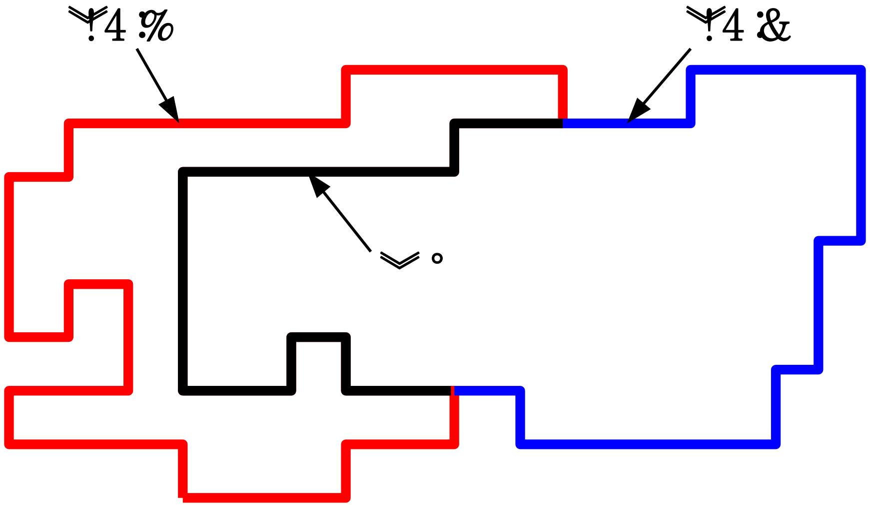


Contour length

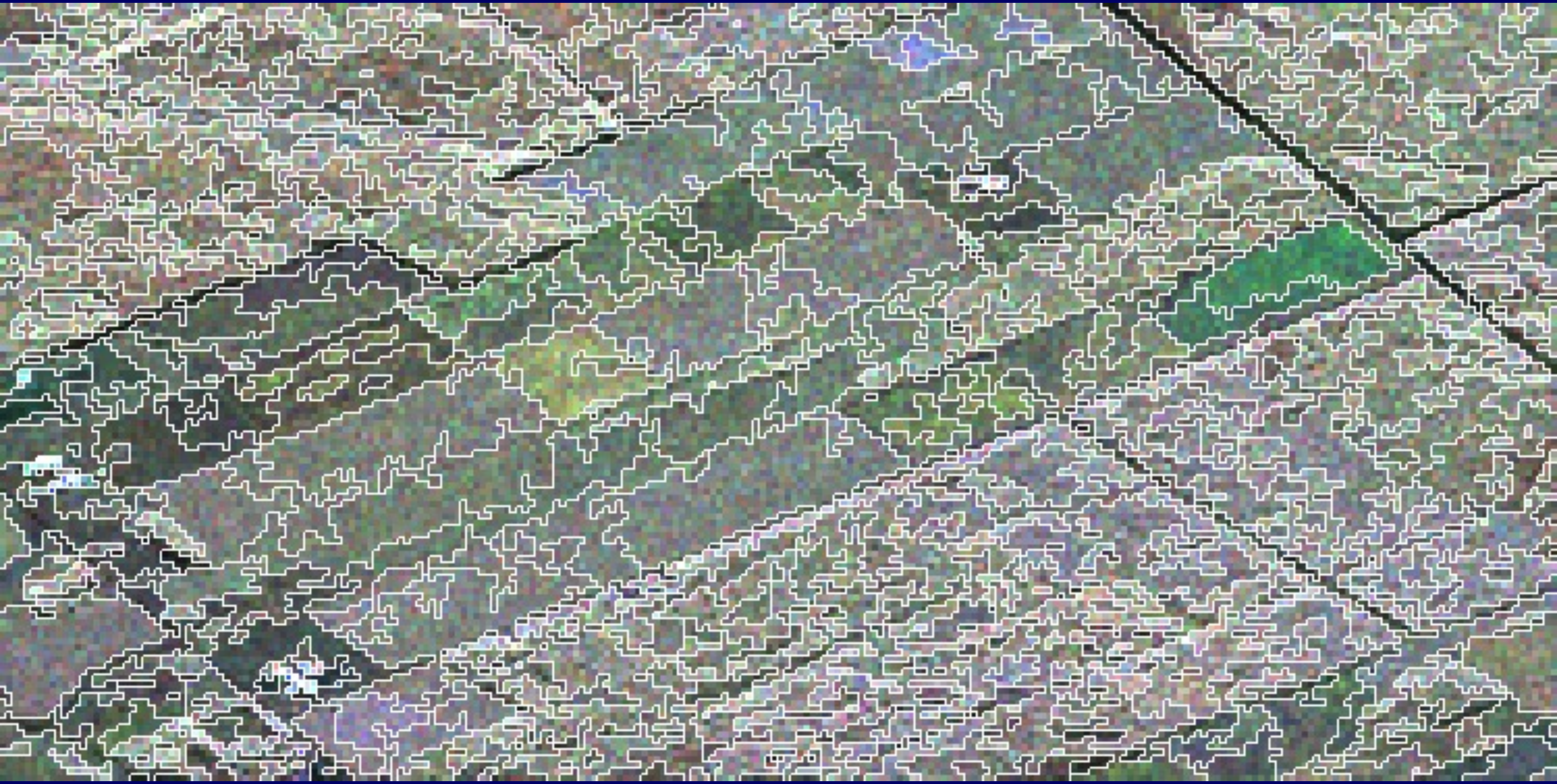
$$Lc \quad = \quad length\ of\ common\ part\ of\ contours$$

$$Lex\,i \quad = \quad length\ of\ exclusive\ part\ for\ S_i$$

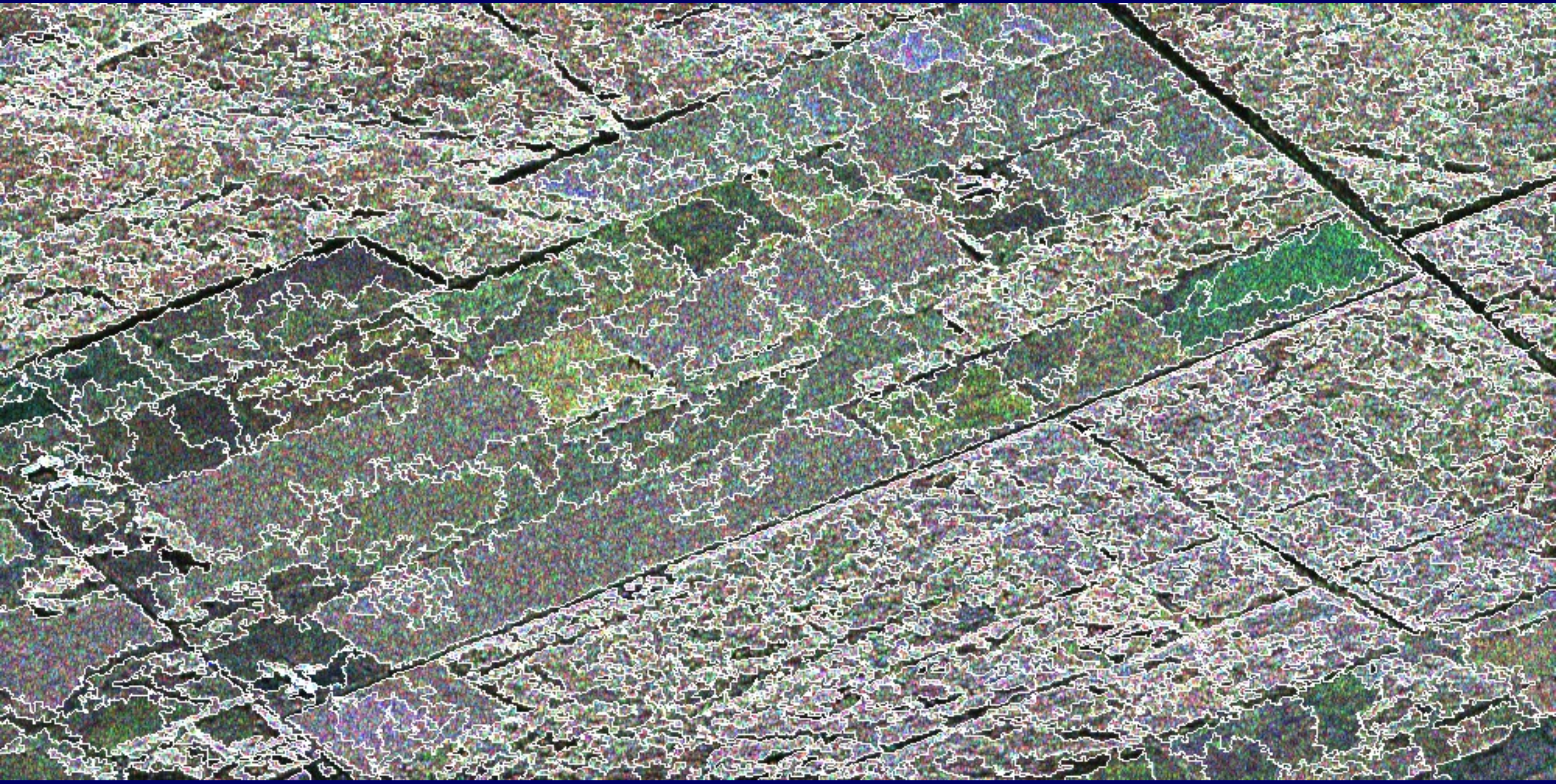
$$Cl \quad = \quad Min\left\{ \frac{Lex\,i}{Lc},\ \frac{Lex\ j}{Lc} \right\}$$



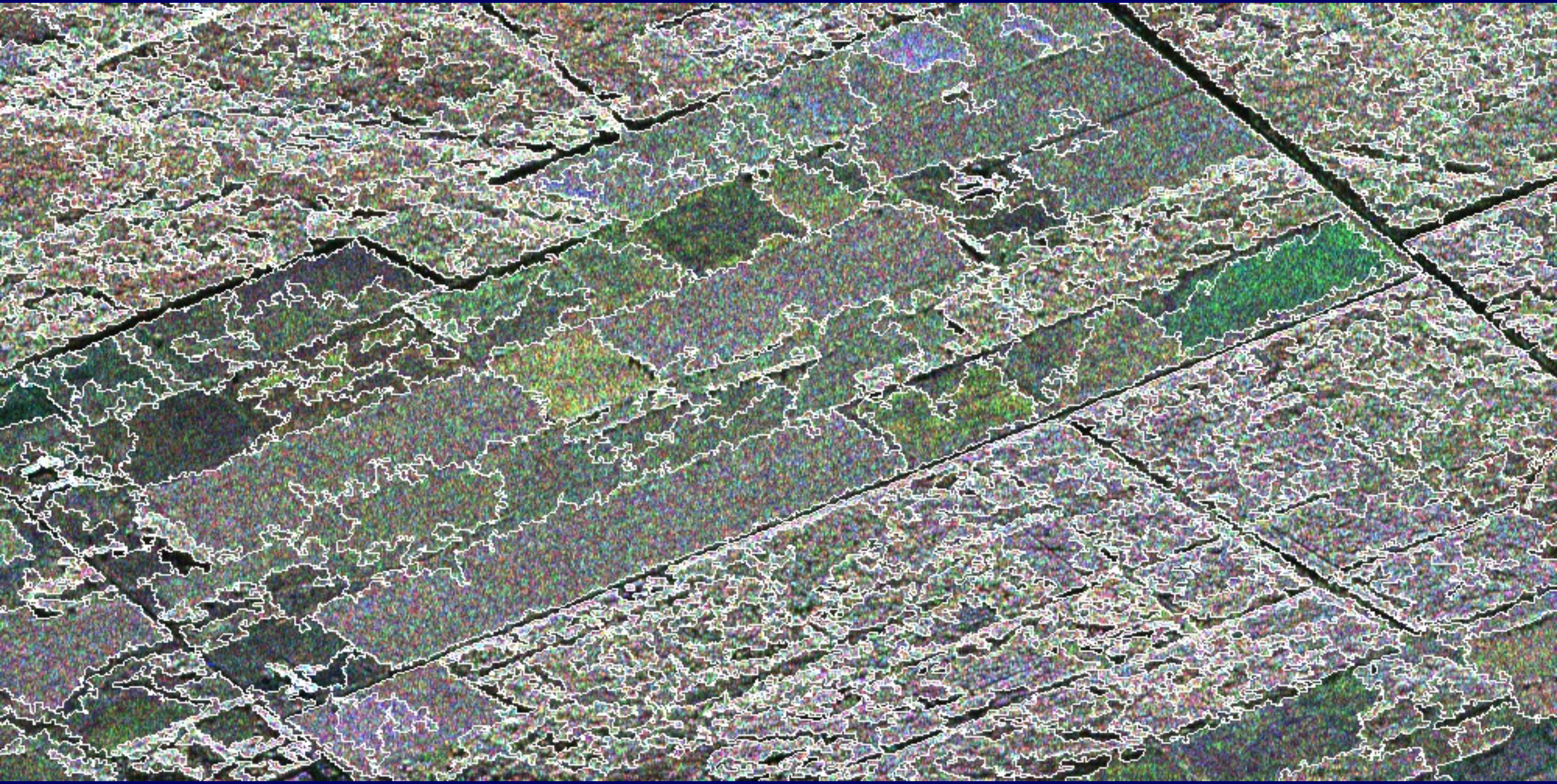
1000 segments – low resolution



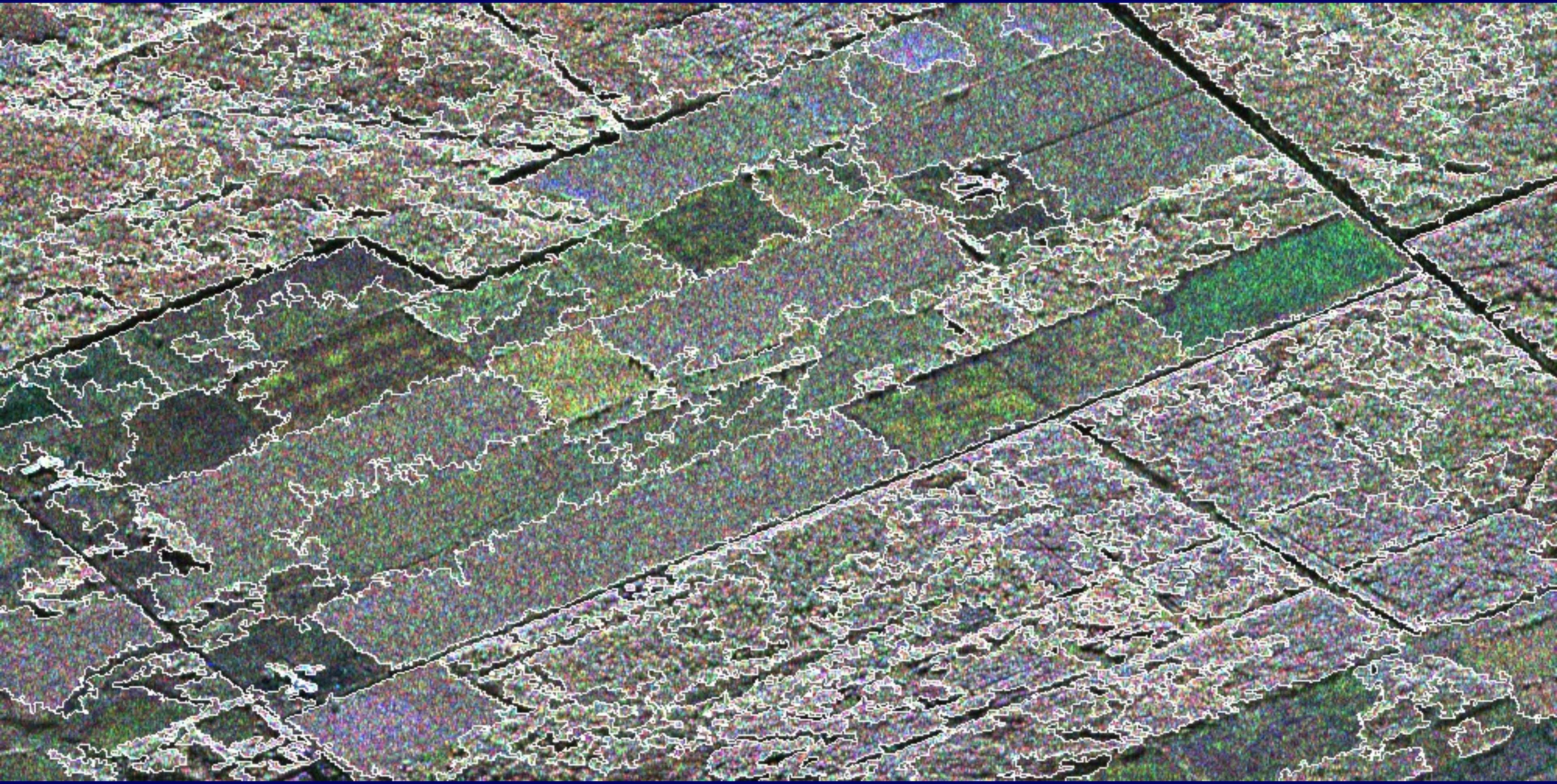
1000 segments



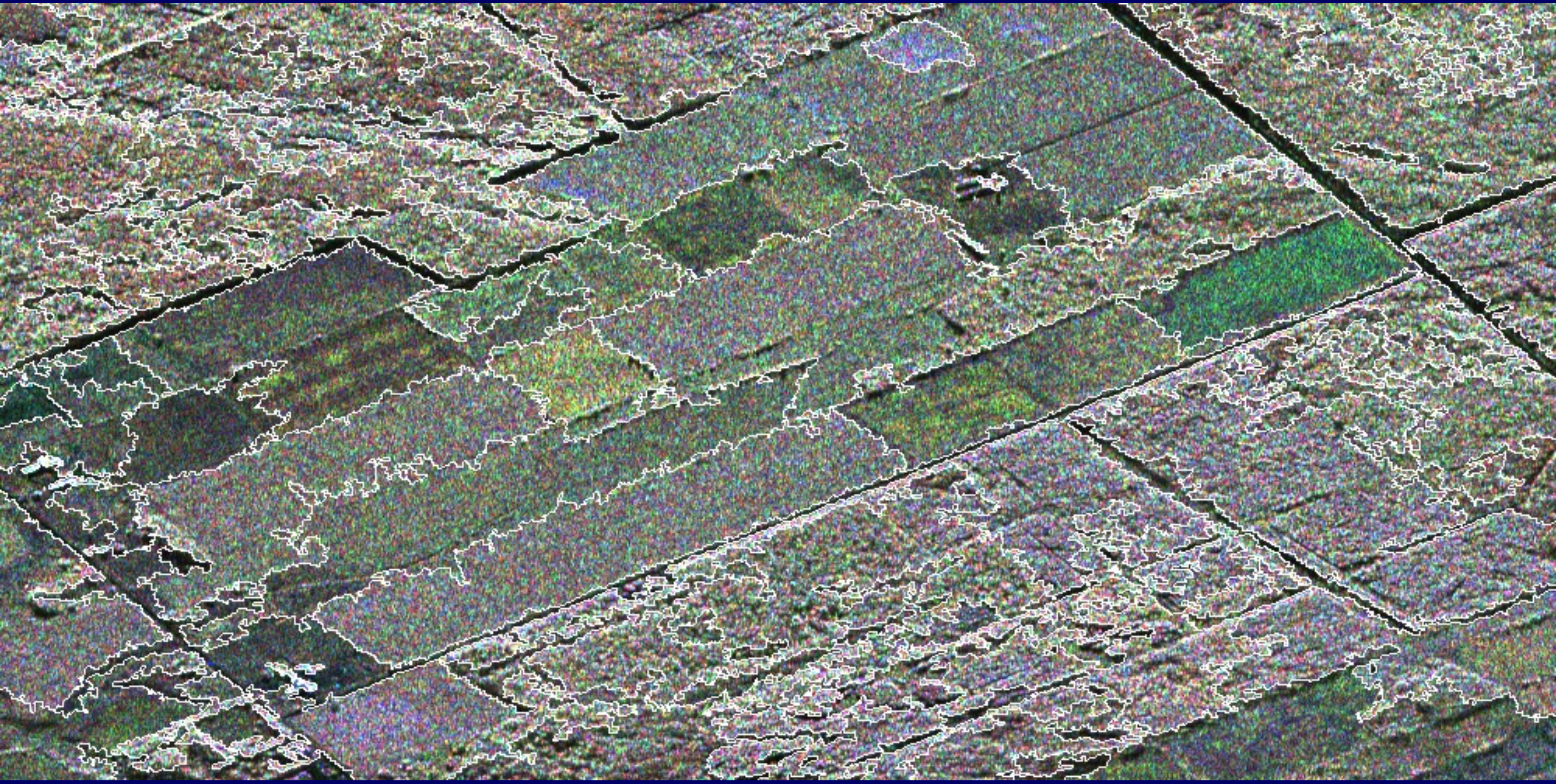
500 segments



200 segments



100 segments



CONCLUSION

- Hierarchical segmentation produces good results
- Criterion should be adapted to the application
- Good polarimetic criterion
- The first merges should be done correctly