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Segmentation of Textured Areas using Polarimetric SAR

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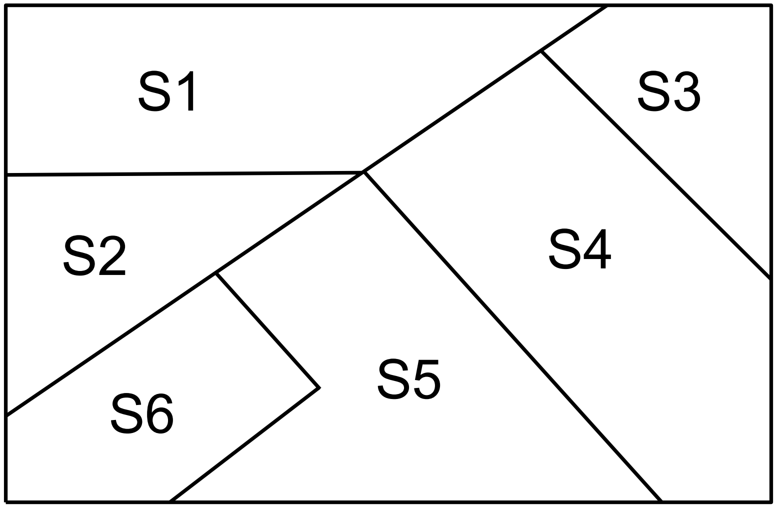
- Hierarchical Image Segmentation
- As a maximum likelihood approximation problem
- Segmentation of polarimetric images
- Segmentation of textured images
- Results

Image Segmentation is the division of the image plane into regions



Two basic questions:

1- **What** kind of regions do we want ?

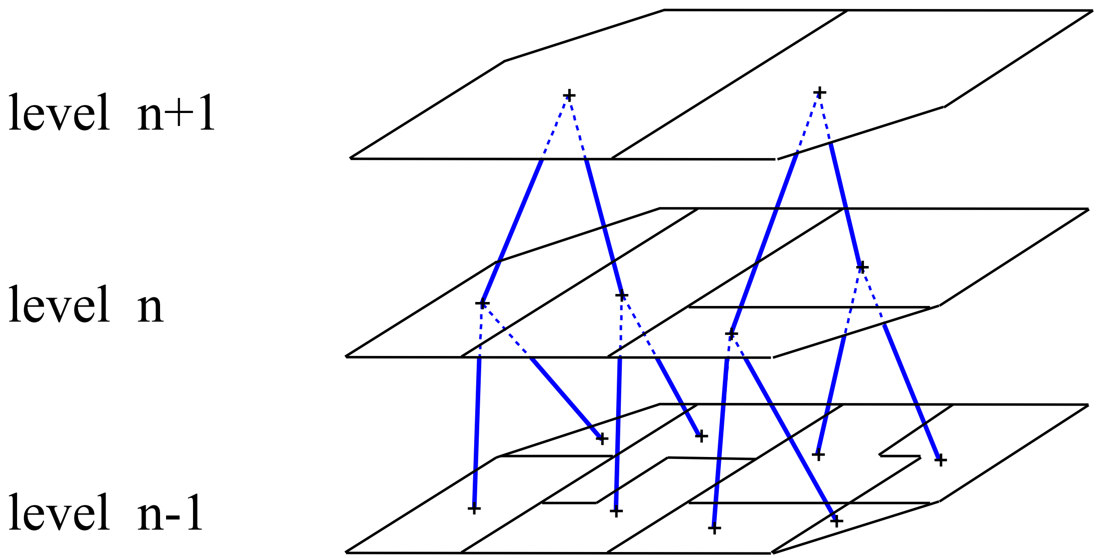
- Homogeneous regions
- Segment similarity

2- **How** can we obtain them ?

- Algorithm design

HIERARCHICAL SEGMENTATION BY STEP-WISE OPTIMISATION

A hierarchical segmentation begins with an initial partition P^0 (with N segments) and then sequentially merges these segments.



Segment tree

SEGMENT SIMILARITY MEASURE

Segmentation → compare two segments

Classification → compare one pixel with one class

Local decision ←→ Global segmentation result

Sequence of tests

SEGMENTATION BY HYPOTHESIS TESTING

Test the similarity of segment covariances $C_i = C_j = C$

- merge segment with same covariance

Use the difference of determinant logarithms as a test statistic

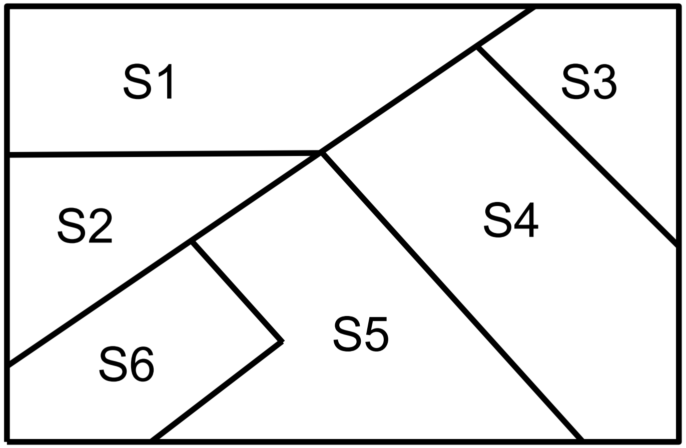
$$C_{i,j} = K\left\{(n_{si} + n_{sj})\ln\left|C_{si \cup sj}\right| - n_{si}\ln\left|C_{si}\right| - n_{sj}\ln\left|C_{sj}\right|\right\}$$

With the scaling factor *K*, the statistic is approximately distributed as a chi-squared variable as n_{si} and n_{sj} become large.

SEGMENTATION AS MAXIMUM LIKELIHOOD APPROXIMATION

1) need a partition of the image

$$P = \{s_k\}, \quad s_k = \{i\} \subset I$$



2) need statistical parameters

$$\theta = \{\theta_s\}, \quad s \in P$$

3) need an image probability model

$$p(x_i \mid \theta_s)$$

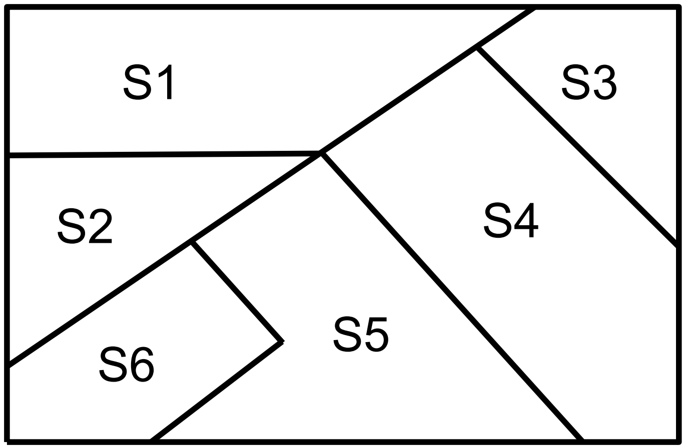
x_i are conditionally independent

Given an image $X = \{x_i\}, \quad i \in I$

the likelihood of $\theta = \{\theta_s\}, \quad P$

is $L(\theta, P \mid X) = p(X \mid \theta, P)$

$$L(\theta, P \mid X) = \prod_{i \in I} p(x_i \mid \theta_{s(i)})\Big|_P$$

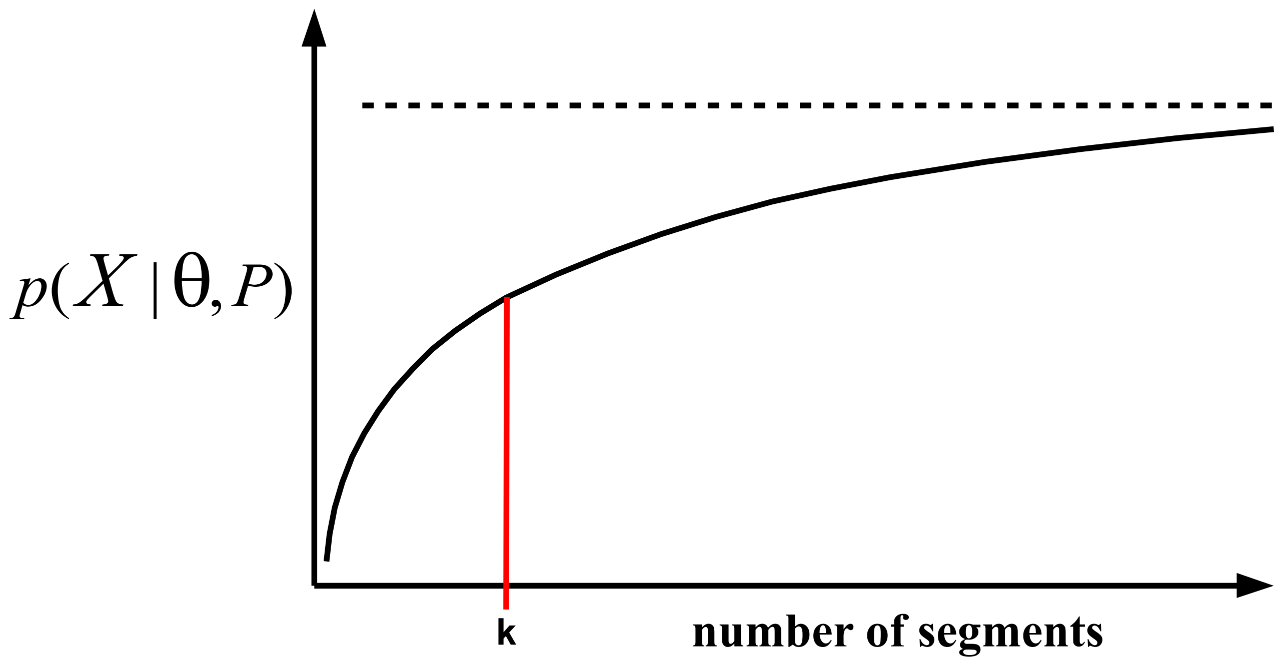


The segmentation problem is to find the partition that maximizes the likelihood.

Global search – too many possible partitions.

θ_s **is derived from statistics calculated over a segment *s*.**

The maximum likelihood increases with the number of segments



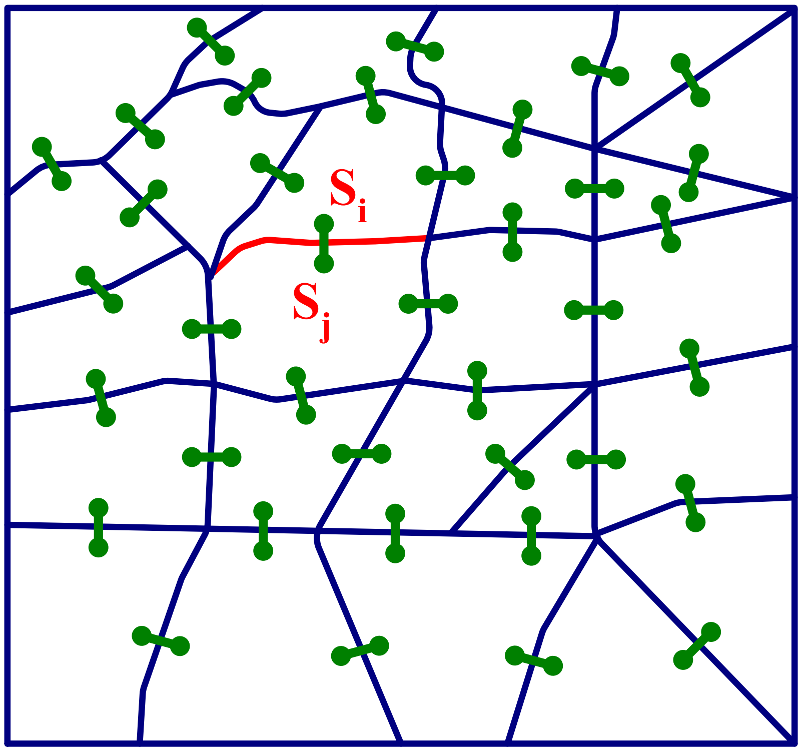
Can't find the optimum partition with k segments, P_k
Too many, except for P_1 and P_{nxn}.

Hierarchical segmentation

→ get P_k from P_{k+1} by merging 2 segments.

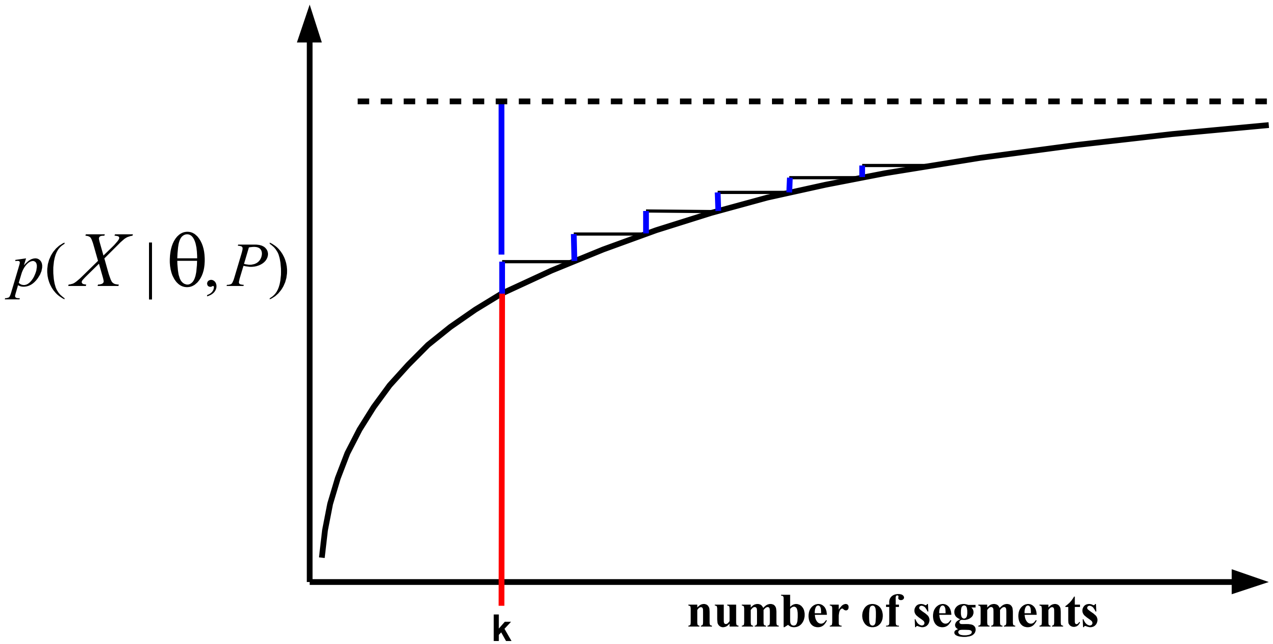
Stepwise optimization

- examine each adjacent segment pair
- merge the pair that minimizes the criterion



Merging criterion:

merge the 2 segments producing the smallest decrease of the maximum likelihood (stepwise optimization)



Sub-optimum within hierarchical merging framework.

Log likelihood form

$$\ln\left(L(\theta, P \mid X)\right) = \ln\left(\prod_{i \in I} p(x_i \mid \theta_{s(i)})\right) = \sum_{i \in I} \ln\left(p(x_i \mid \theta_{s(i)})\right)$$

Summation inside region

$$\sum_{s \in P} \sum_{i \in s} \ln\left(p(x_i \mid \theta_s)\right) = \sum_{s \in P} LML(s)$$

Criterion → cost of merging 2 segments

$$\Delta = LML(s_i) + LML(s_j) - LML(s_i \cup s_j)$$

$$\Delta = \sum_{x \in s_i} \ln\left(p(x \mid \theta_{s_i})\right) + \sum_{x \in s_j} \ln\left(p(x \mid \theta_{s_j})\right) - \sum_{x \in s_i \cup s_j} \ln\left(p(x \mid \theta_{s_i \cup s_j})\right)$$

minimize $|\Delta|$

POLARIMETRIC SAR IMAGE

Multi-channel image – 3 complex elements

$$x = \begin{bmatrix} hh \\ hv \\ vv \end{bmatrix}$$

each element has
a zero mean circular
gaussian distribution

Complex gaussian pdf (Σ is the covariance matrix)

$$p(x \mid \Sigma) = \frac{1}{\pi^3 |\Sigma|} \exp\left(-x^* \Sigma^{-1} x\right)$$

x^* is the complex conjugate transpose of x

The best maximum likelihood estimate of Σ is the covariance calculated over the region (segment)

$$\hat{\Sigma} = C = \frac{1}{n_s} \sum_{x \in s} x\, x^*$$

n_s is the number of pixels in segment s

$$\boldsymbol{C} = \frac{1}{n} \begin{bmatrix} \sum hh\, hh^* & \sum hh\, hv^* & \sum hh\, vv^* \\ \sum hv\, hh^* & \sum hv\, hv^* & \sum hv\, vv^* \\ \sum vv\, hh^* & \sum vv\, hv^* & \sum vv\, vv^* \end{bmatrix}$$

***LML* for a region *s* is**

$$LML(s)=\sum_{x\in s}\ln\left(p(x\,|\,C_s)\right)=\sum_{x\in s}\ln\left(\frac{1}{\pi^3|C_s|}\exp\left(-x^*C_s^{-1}x\right)\right)$$

$$=\sum_{x\in s}\left[-\ln\pi^3-\ln|C_s|-x^*C_s^{-1}x\right]$$

$$=-n_s\ln\pi^3-n_s\ln|C_s|-\sum_{x\in s}x^*C_s^{-1}x$$

$$=-n_s\ln|C_s|-n_s\ln\pi^3-3n_s$$

constant term for the whole image

The variation produced by merging 2 segments is

$$\Delta = LML(s_i) + LML(s_j) - LML(s_i \cup s_j)$$
$$= -n_{si} \ln\left|C_{si}\right| - n_{sj} \ln\left|C_{sj}\right| + (n_{si} + n_{sj}) \ln\left|C_{si \cup sj}\right|$$

Hierarchical segmentation:

at each iteration, merge the 2 segments that minimize the stepwise criterion $C_{i,j}$

$$C_{i,j} = (n_{si} + n_{sj}) \ln\left|C_{si \cup sj}\right| - n_{si} \ln\left|C_{si}\right| - n_{sj} \ln\left|C_{sj}\right|$$

MULTILOOK IMAGE

For *L*-look image, a pixel *k* should be represented by its *L*-look covariance matrix, Z_k

Z_k follows a complex Wishart distribution

$$p(Z_k \mid \Sigma) = \frac{L^{3L} \left| Z_k \right|^{L-3} \exp\left\{ -L \, tr\left(\Sigma^{-1} Z_k \right) \right\}}{\pi^3 \, \Gamma(L)\Gamma(L-1)\Gamma(L-2) \left| \Sigma \right|^{L}}$$

The variation produced by merging 2 segments is

$$\Delta = MLL(S_i) + MLL(S_j) - MLL(S_i \cup S_j)$$
$$= L(m_i + m_j)\ln\left|C_{Si \cup Sj}\right| - Lm_i \ln\left|C_{Si}\right| - Lm_j \ln\left|C_{Sj}\right|.$$

This is equivalent to the previous criterion
where $n = L\,m$ *(m is the number of L-look pixels)*

$$C_{i,j} = (n_{si} + n_{sj})\ln\left|C_{si \cup sj}\right| - n_{si}\ln\left|C_{si}\right| - n_{sj}\ln\left|C_{sj}\right|$$

TEXTURED IMAGE

Assume that a texture value μ modifies the covariance matrix

$$\boldsymbol{Z}_{\mathbf{k}} = \mu_{\mathbf{k}}\, \boldsymbol{Z}_{\mathbf{k\text{-}homogeneous}}$$

$\boldsymbol{Z}_{\mathbf{k}}$ follows a K distribution

$$p(Z_k \mid \alpha, \Sigma) = \frac{(\alpha L)^{(3L+\alpha)/2}\; 2\left|Z_k\right|^{L-3} \left(tr\left(\Sigma^{-1} Z_k\right)\right)^{(\alpha-3L)/2}}{\pi^3\; \Gamma(L)\Gamma(L-1)\Gamma(L-2)\; \Gamma(\alpha) \left|\Sigma\right|^{L}}$$

$$K_{3L-\alpha}\left\{2\sqrt{\alpha L\; tr\left(\Sigma^{-1} Z_k\right)}\right\}$$

The maximum log likelihood for one segment is

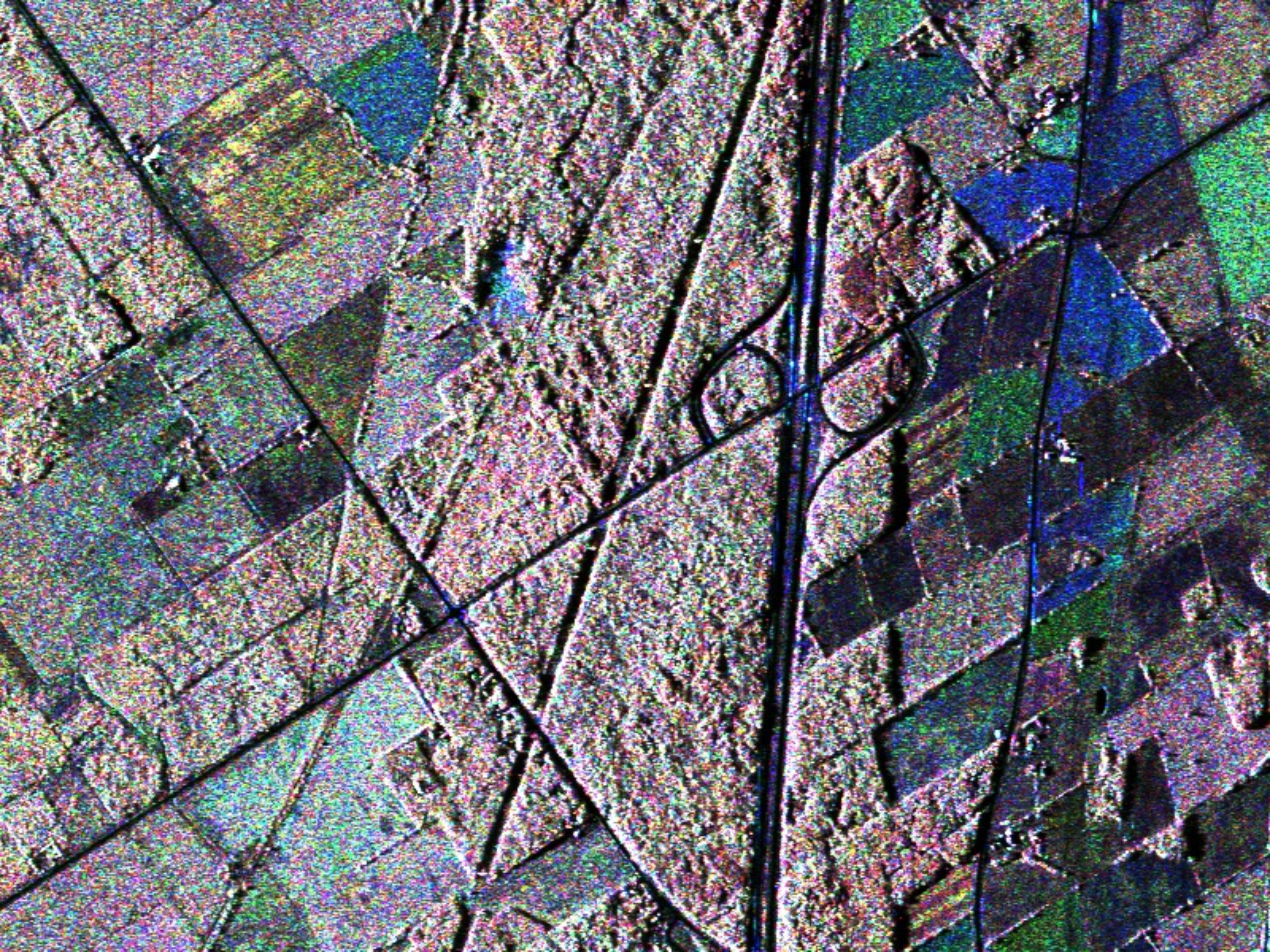
$$MLL(S) \simeq n\,\tfrac{3L+\alpha}{2}\ln(\alpha L) - n\ln(\Gamma(\alpha)) - nL\ln(|\Sigma|)$$
$$+\tfrac{\alpha-3L}{2}\sum_{k\in S}\ln\left(tr\left(\Sigma^{-1}Z_k\right)\right)$$
$$+\sum_{k\in S}K_{3L-\alpha}\left\{2\sqrt{\alpha L\, tr\left(\Sigma^{-1}Z_k\right)}\right\}$$

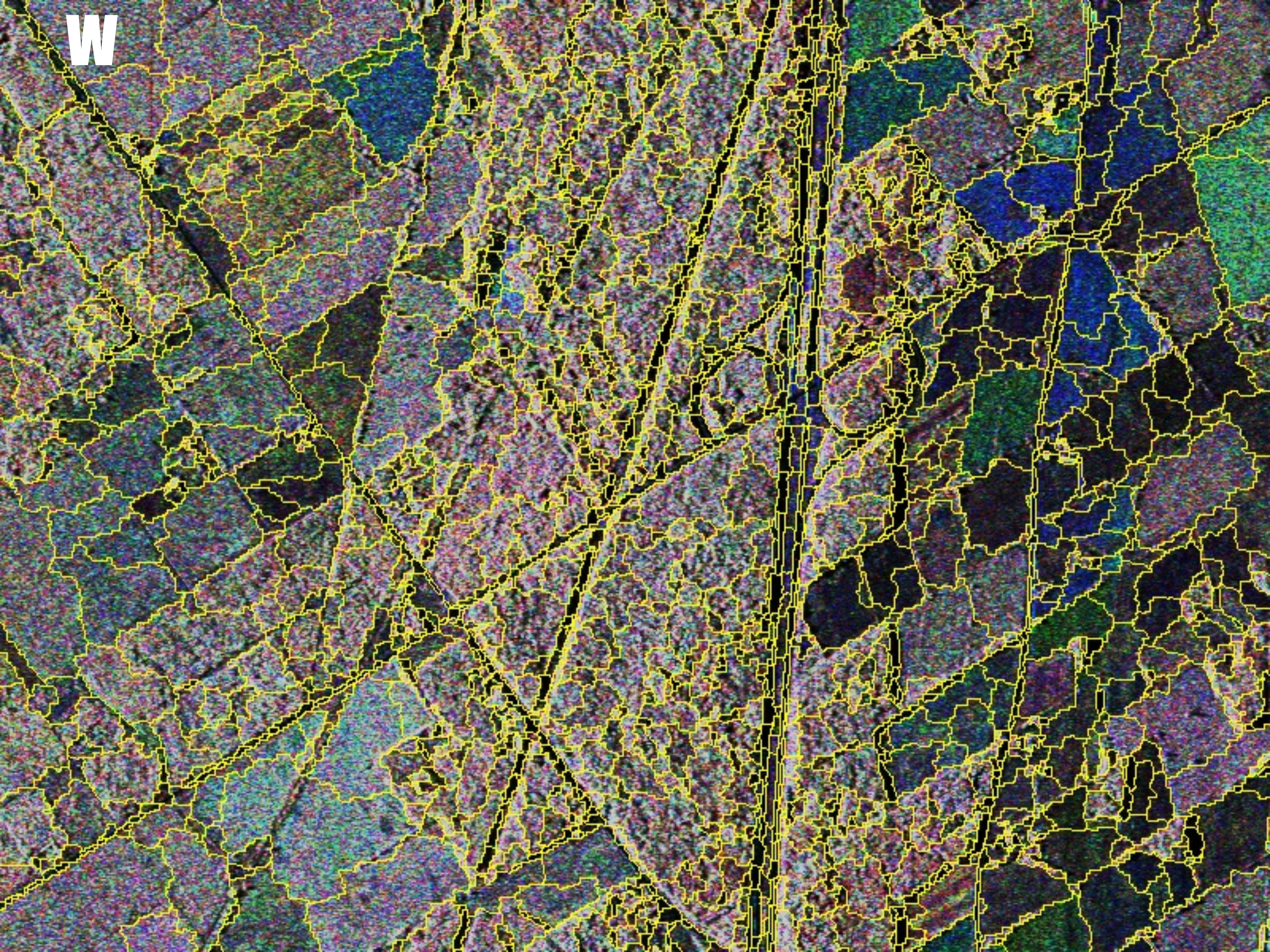
Best α and Σ → Iteration (gradient descent)

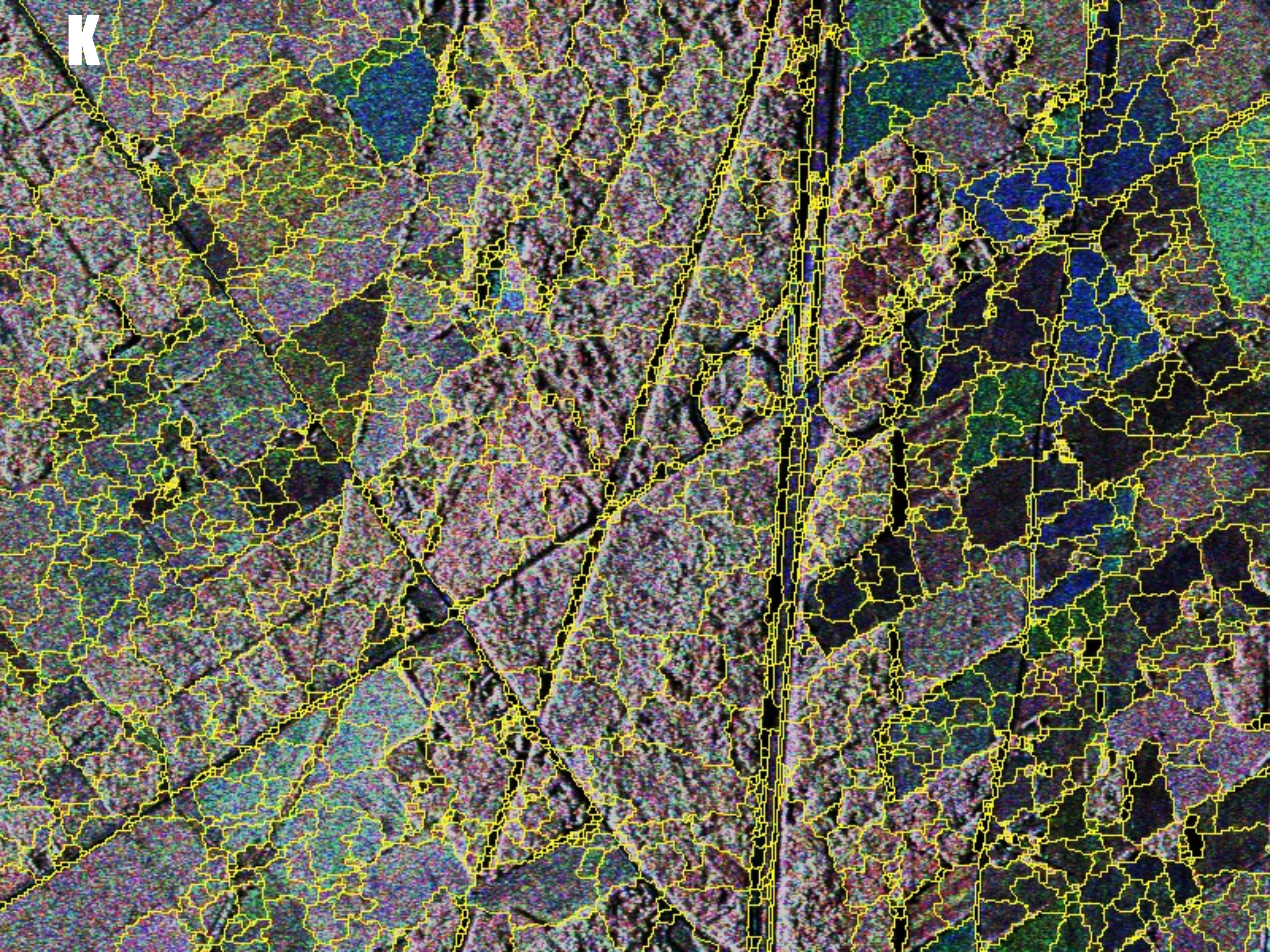
Approximation

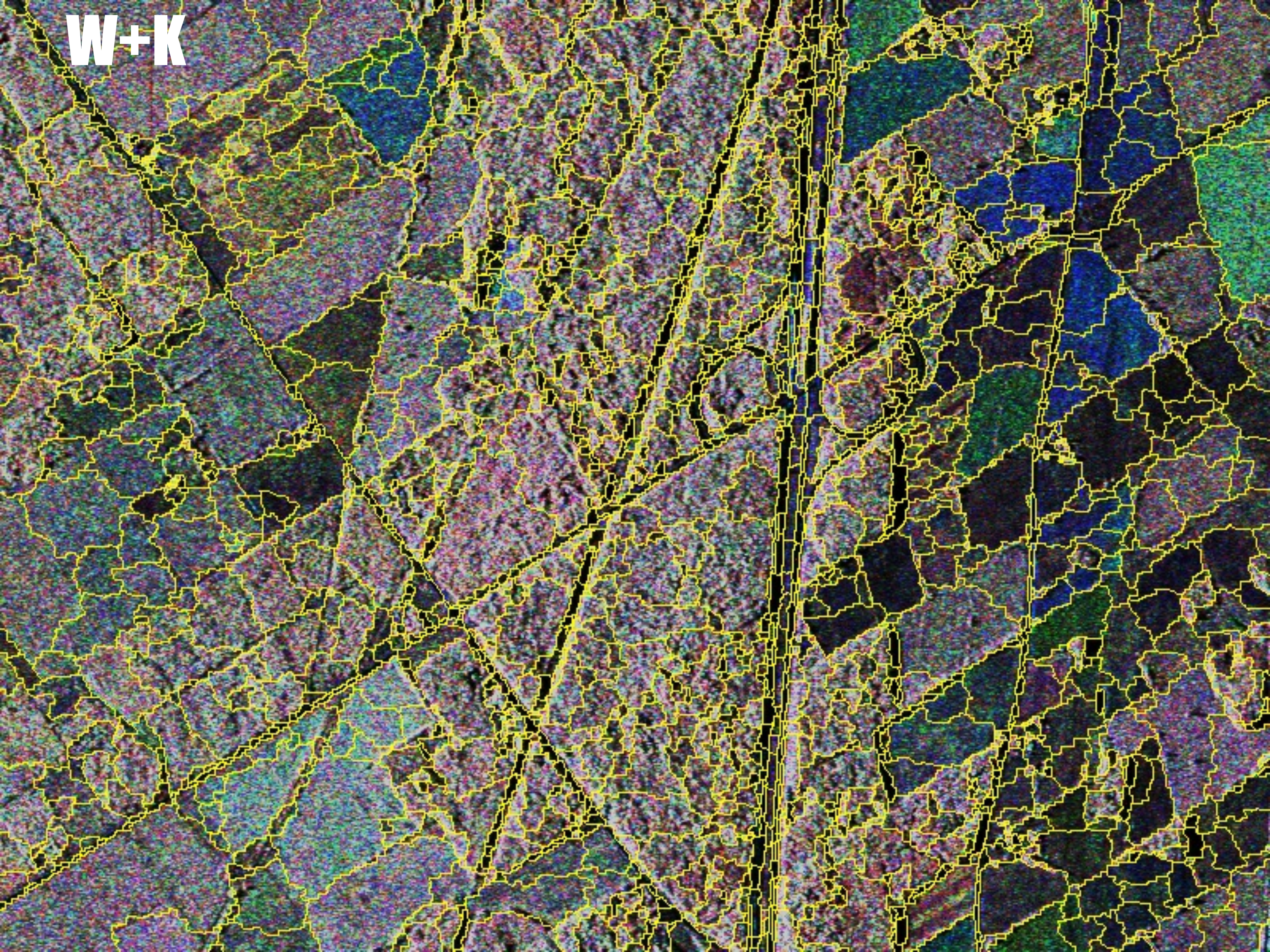
Σ = segment covariance matrix

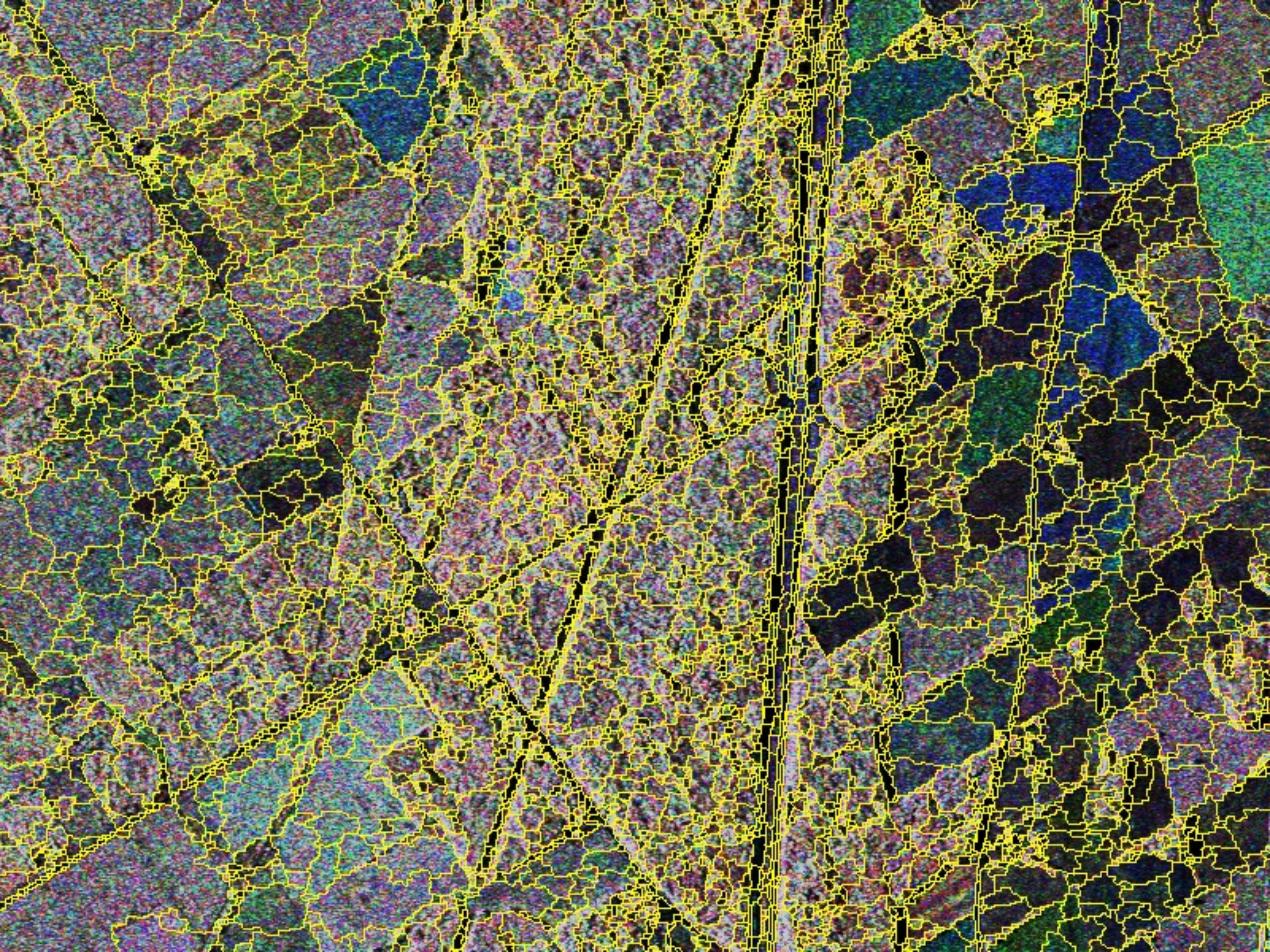
$\alpha = 1/(CV_R)^2$ → Method of Moments

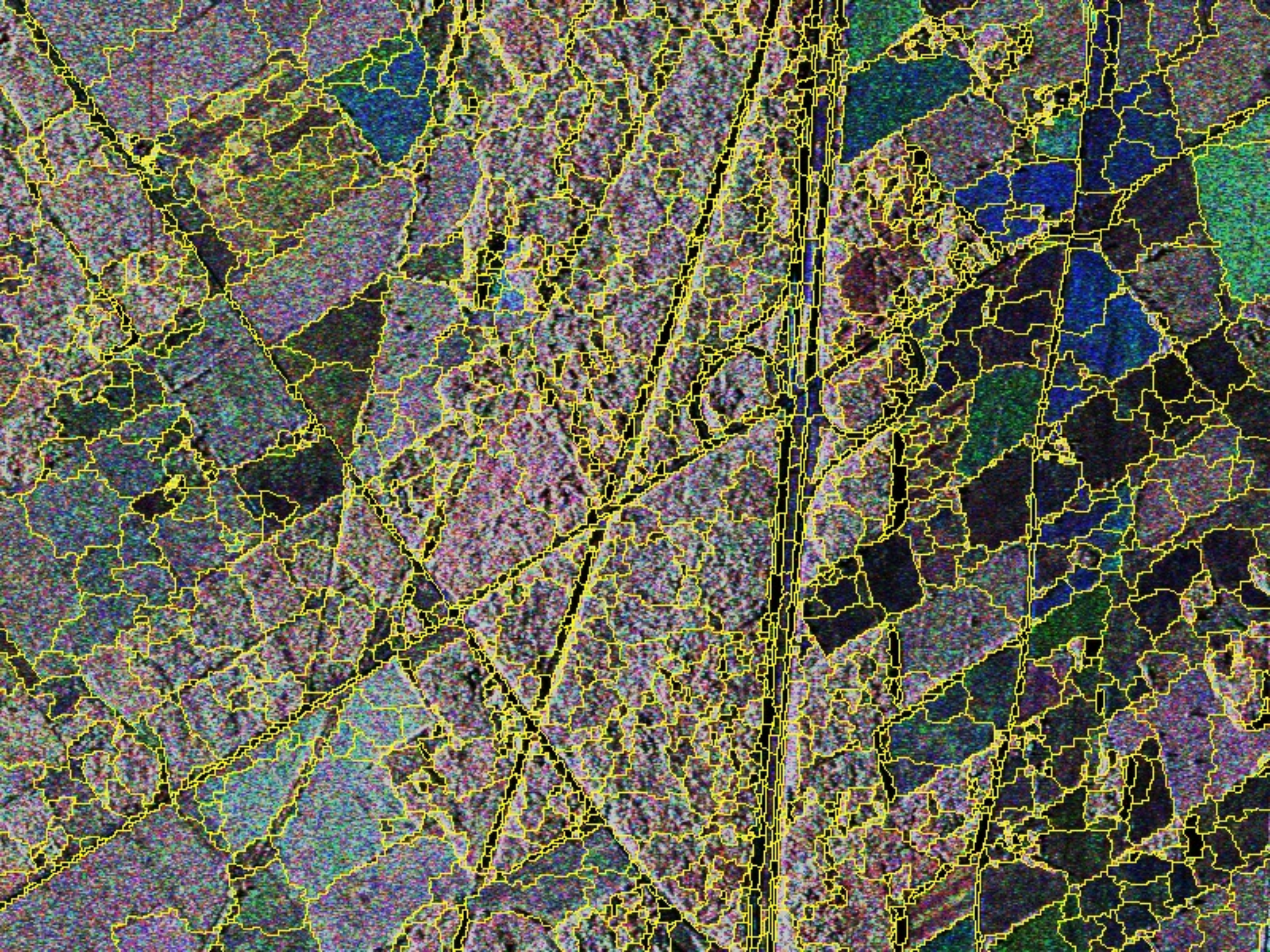


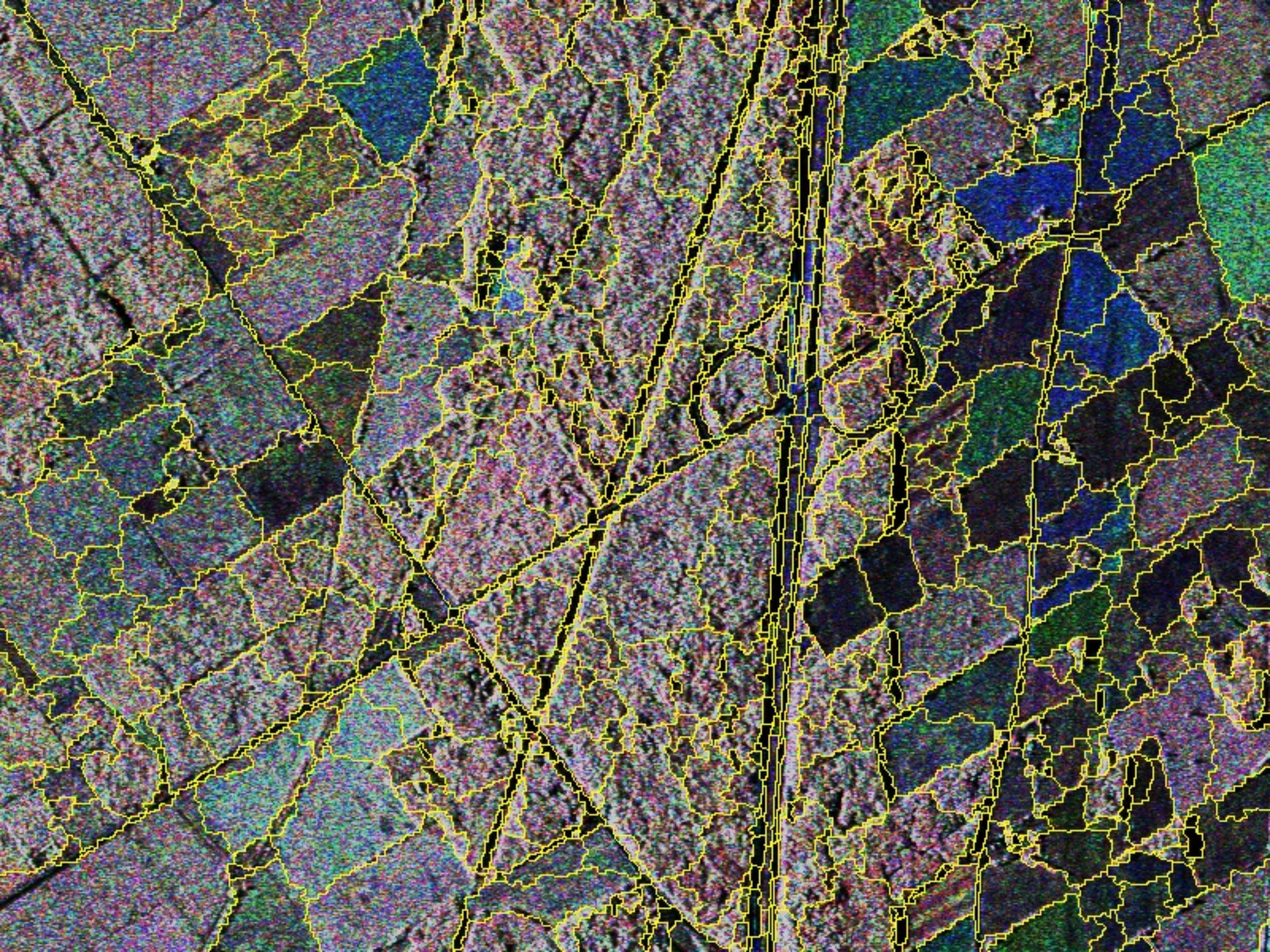

W

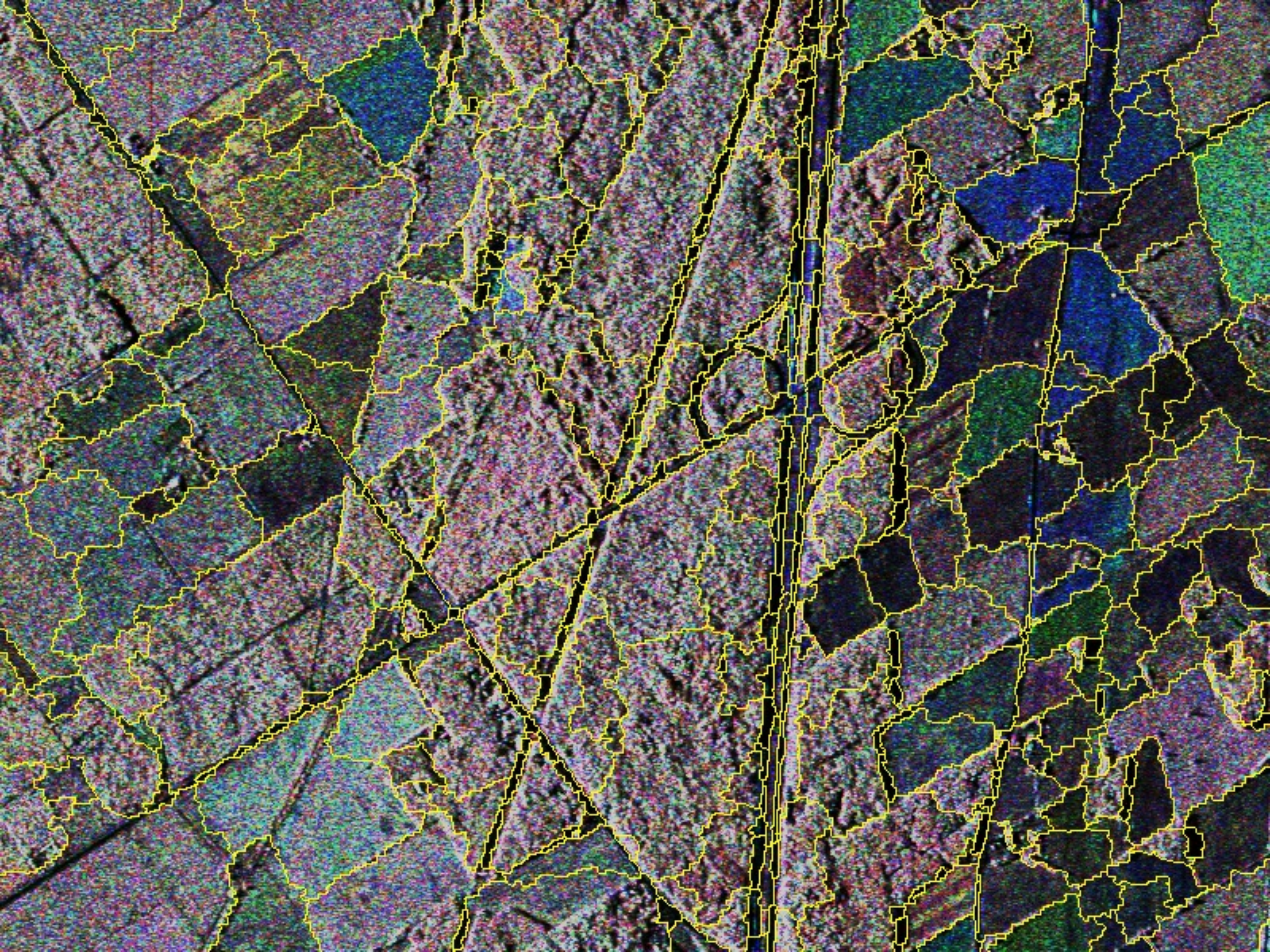

K


W+K









SEGMENT SHAPE CRITERIA

High speckle noise

→ first merges produce ill formed segments

- Bonding box – perimeter Cp
- Bonding box – area Ca
- Contour length Cl

New criteria

$$C_{i,j}^{contour} = C_{i,j}^{polar} \times Cp^2 \times Ca \times Cl$$

CONCLUSION

- Hierarchical segmentation produces good results
- Good polarimetric criteria for homogeneous and textured fields
- Shape criteria are useful