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Segmentation of polarimetric SAR images: a best estimate partitioning approach

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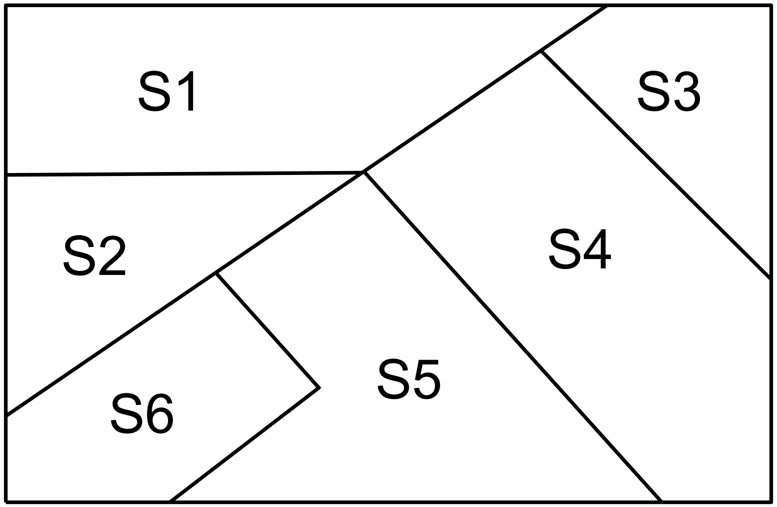
Segmentation of polarimetric SAR images: a best estimate partitioning approach

- Hierarchical Image Segmentation
- As a maximum likelihood approximation problem
- Segmentation of polarimetric images
- Segmentation of textured images
- Results

Image Segmentation is the division of the image plane into regions



Two basic questions:

1- **What** kind of regions do we want ?

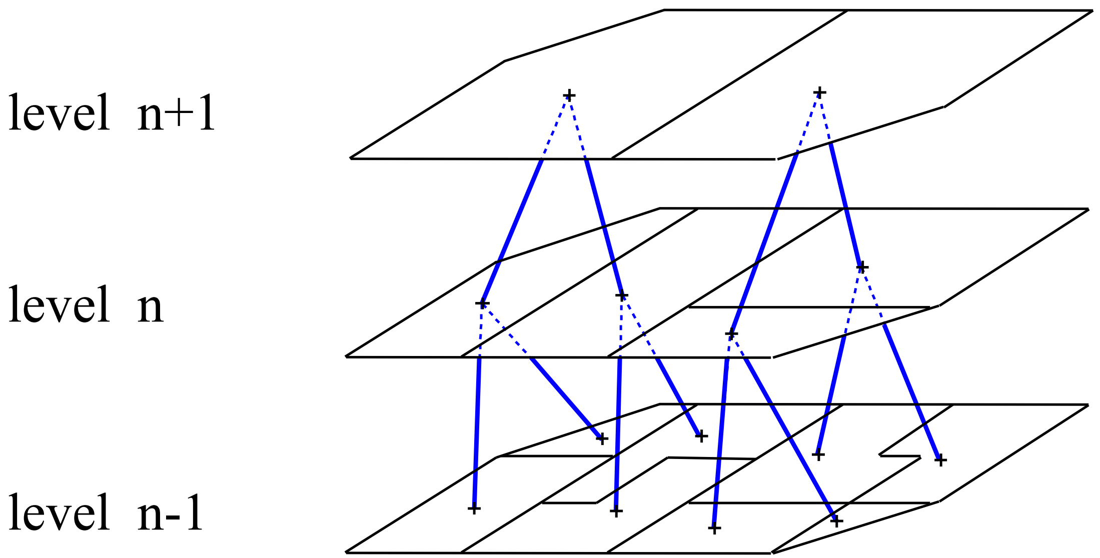
- Homogeneous regions
- Segment similarity

2- **How** can we obtain them ?

- Algorithm design

HIERARCHICAL SEGMENTATION BY STEP-WISE OPTIMISATION

A hierarchical segmentation begins with an initial partition P^0 (with N segments) and then sequentially merges these segments.



Segment tree

SEGMENT SIMILARITY MEASURE

Segmentation → compare two segments

Classification → compare one pixel with one class

Local decision ←→ Global segmentation result

Sequence of tests

SEGMENTATION BY HYPOTHESIS TESTING

Test the similarity of segment covariances $C_i = C_j = C$

- merge segment with same covariance

Use the difference of determinant logarithms as a test statistic

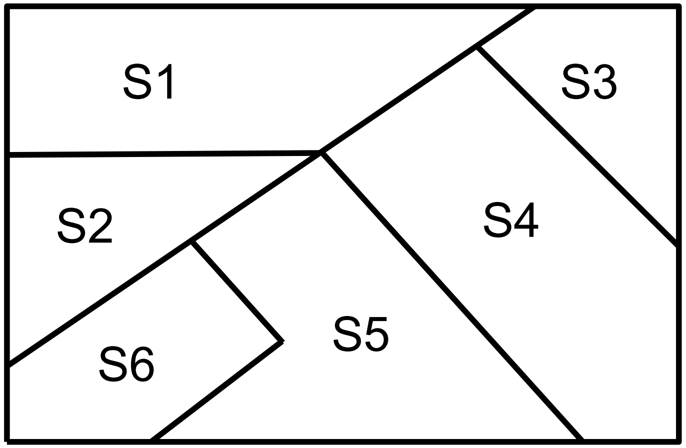
$$C_{i,j} = K\left\{(n_{si} + n_{sj})\ln\left|C_{si\cup sj}\right| - n_{si}\ln\left|C_{si}\right| - n_{sj}\ln\left|C_{sj}\right|\right\}$$

With the scaling factor *K*, the statistic is approximately distributed as a chi-squared variable as n_{si} and n_{sj} become large.

SEGMENTATION AS MAXIMUM LIKELIHOOD APPROXIMATION

1) need a partition of the image

$$P = \{s_k\}, \quad s_k = \{i\} \subset I$$



2) need statistical parameters

$$\theta = \{\theta_s\}, \quad s \in P$$

3) need an image probability model

$$p(x_i \mid \theta_s)$$

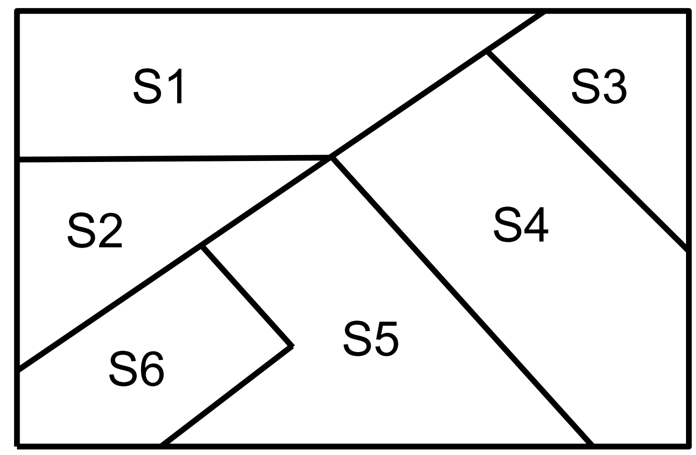
x_i are conditionally independent

Given an image $X = \{x_i\}, \quad i \in I$

the likelihood of $\theta = \{\theta_s\}, \quad P$

is $L(\theta, P \mid X) = p(X \mid \theta, P)$

$$L(\theta, P \mid X) = \prod_{i \in I} p(x_i \mid \theta_{s(i)}) \Big|_P$$

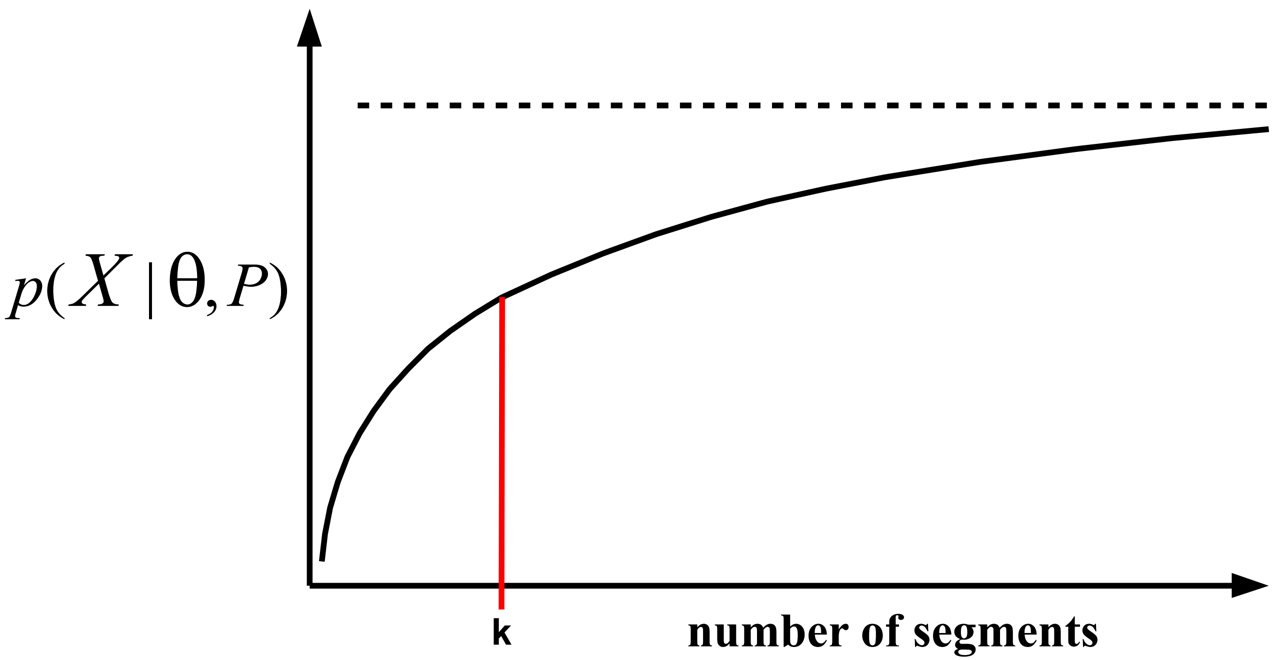


The segmentation problem is to find the partition that maximizes the likelihood.

Global search – too many possible partitions.

θ_s **is derived from statistics calculated over a segment *s*.**

The maximum likelihood increases with the number of segments

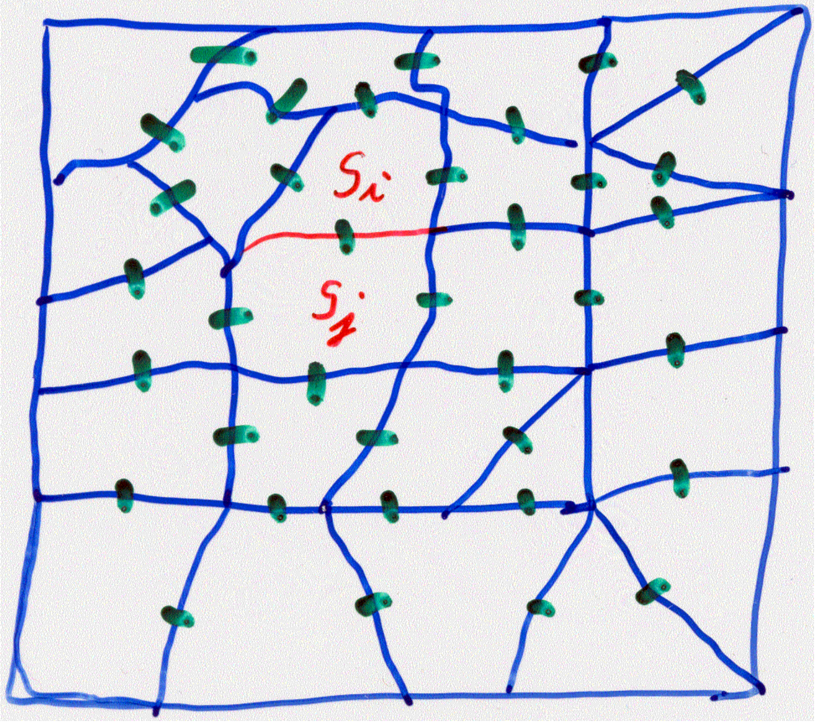


Can't find the optimum partition with k segments, P_k
Too many, except for P_1 and P_{nxn}.

Hierarchical segmentation
→ get P_k from P_{k+1} by merging 2 segments.

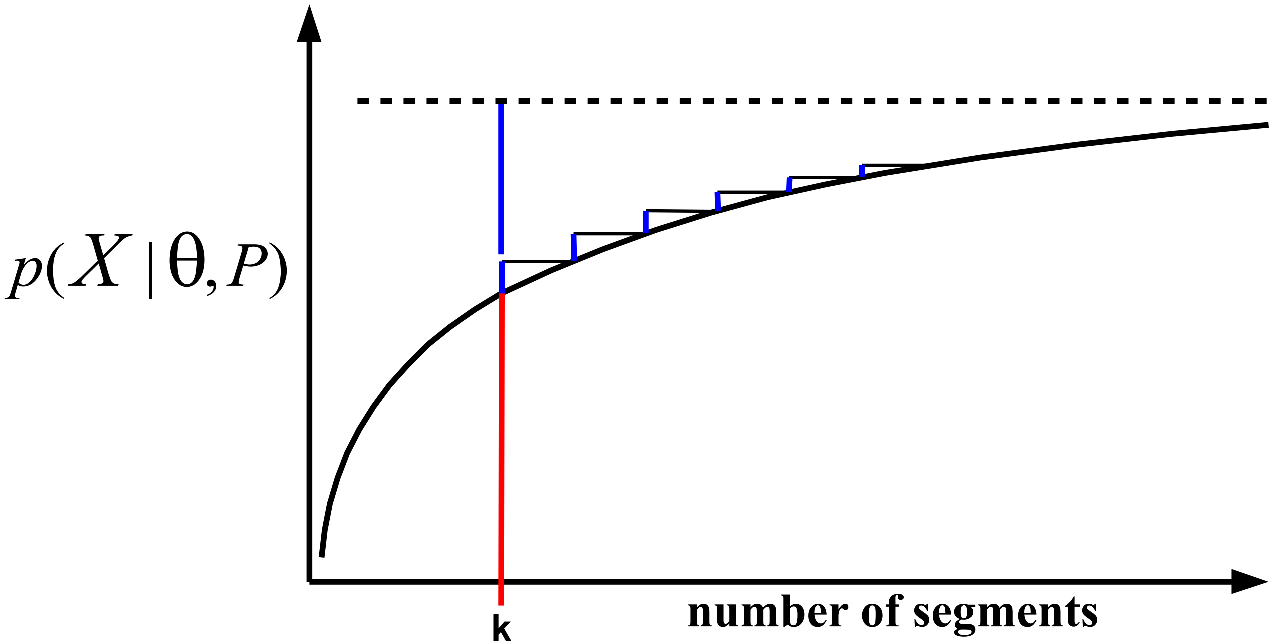
Stepwise optimization

- examine each adjacent segment pair
- merge the pair that minimizes the criterion



Merging criterion:

merge the 2 segments producing the smallest decrease of the maximum likelihood (stepwise optimization)



Sub-optimum within hierarchical merging framework.

Log likelihood form

$$\ln\left(L(\theta, P \mid X)\right) = \ln\left(\prod_{i \in I} p(x_i \mid \theta_{s(i)})\right) = \sum_{i \in I} \ln\left(p(x_i \mid \theta_{s(i)})\right)$$

Summation inside region

$$\sum_{s \in P} \sum_{i \in s} \ln\left(p(x_i \mid \theta_s)\right) = \sum_{s \in P} LML(s)$$

Criterion → cost of merging 2 segments

$$\Delta = LML(s_i) + LML(s_j) - LML(s_i \cup s_j)$$

$$\Delta = \sum_{x \in s_i} \ln\left(p(x \mid \theta_{s_i})\right) + \sum_{x \in s_j} \ln\left(p(x \mid \theta_{s_j})\right) - \sum_{x \in s_i \cup s_j} \ln\left(p(x \mid \theta_{s_i \cup s_j})\right)$$

minimize $|\Delta|$

POLARIMETRIC SAR IMAGE

Multi-channel image – 3 complex elements

$$x = \begin{bmatrix} hh \\ hv \\ vv \end{bmatrix}$$

each element has
a zero mean circular
gaussian distribution

Complex gaussian pdf (Σ is the covariance matrix)

$$p(x \mid \Sigma) = \frac{1}{\pi^3 \left|\Sigma\right|} \exp\left(-x^* \Sigma^{-1} x\right)$$

x^* is the complex conjugate transpose of x

The best maximum likelihood estimate of Σ is the covariance calculated over the region (segment)

$$\hat{\Sigma} = C = \frac{1}{n_s} \sum_{x \in s} x\, x^*$$

n_s is the number of pixels in segment s

$$\boldsymbol{C} = \frac{1}{n} \begin{bmatrix} \sum hh\, hh^* & \sum hh\, hv^* & \sum hh\, vv^* \\ \sum hv\, hh^* & \sum hv\, hv^* & \sum hv\, vv^* \\ \sum vv\, hh^* & \sum vv\, hv^* & \sum vv\, vv^* \end{bmatrix}$$

LML **for a region *s* is**

$$LML(s) = \sum_{x \in s} \ln\left(p(x \mid C_s)\right) = \sum_{x \in s} \ln\left(\frac{1}{\pi^3 |C_s|} \exp\left(-x^* C_s^{-1} x\right)\right)$$

$$= \sum_{x \in s}\left[-\ln \pi^3 - \ln|C_s| - x^* C_s^{-1} x\right]$$

$$= -n_s \ln \pi^3 - n_s \ln|C_s| - \sum_{x \in s} x^* C_s^{-1} x$$

$$= -n_s \ln|C_s| - n_s \ln \pi^3 - 3n_s$$

constant term for the whole image

The variation produced by merging 2 segments is

$$\Delta = LML(s_i) + LML(s_j) - LML(s_i \cup s_j)$$
$$= -n_{si} \ln\left|C_{si}\right| - n_{sj} \ln\left|C_{sj}\right| + (n_{si} + n_{sj}) \ln\left|C_{si \cup sj}\right|$$

Hierarchical segmentation:

at each iteration, merge the 2 segments that minimize the stepwise criterion $C_{i,j}$

$$C_{i,j} = (n_{si} + n_{sj}) \ln\left|C_{si \cup sj}\right| - n_{si} \ln\left|C_{si}\right| - n_{sj} \ln\left|C_{sj}\right|$$

MULTILOOK IMAGE

For *L*-look image, a pixel *k* should be represented by its *L*-look covariance matrix, Z_k

Z_k follows a complex Wishart distribution

$$p(Z_k \mid \Sigma) = \frac{L^{3L} \left|Z_k\right|^{L-3} \quad \exp\left\{-L\, tr\left(\Sigma^{-1} Z_k\right)\right\}}{\pi^3\, \Gamma(L)\Gamma(L-1)\Gamma(L-2) \left|\Sigma\right|^{L}}$$

The variation produced by merging 2 segments is

$$\Delta = MLL(S_i) + MLL(S_j) - MLL(S_i \cup S_j)$$
$$= L(m_i + m_j)\ln\left|C_{Si\cup Sj}\right| - Lm_i \ln\left|C_{Si}\right| - Lm_j \ln\left|C_{Sj}\right|.$$

This is equivalent to the previous criterion
where $n = L\,m$ *(m is the number of L-look pixels)*

$$C_{i,j} = (n_{si} + n_{sj})\ln\left|C_{si\cup sj}\right| - n_{si}\ln\left|C_{si}\right| - n_{sj}\ln\left|C_{sj}\right|$$

TEXTURED IMAGE

Assume that a texture value μ modifies the covariance matrix

$$\boldsymbol{Z}_{\mathbf{k}} = \mu_{\mathbf{k}}\, \boldsymbol{Z}_{\mathbf{k\text{-}homogeneous}}$$

$\boldsymbol{Z}_{\mathbf{k}}$ **follows a K distribution**

$$p(Z_k \mid \alpha, \Sigma) = \frac{(\alpha L)^{(3L+\alpha)/2}\, 2\left|Z_k\right|^{L-3} \left(tr\left(\Sigma^{-1} Z_k\right)\right)^{(\alpha-3L)/2}}{\pi^3\, \Gamma(L)\Gamma(L-1)\Gamma(L-2)\,\Gamma(\alpha)\left|\Sigma\right|^{L}}$$

$$K_{3L-\alpha}\left\{2\sqrt{\alpha L\, tr\left(\Sigma^{-1} Z_k\right)}\right\}$$

The maximum log likelihood for one segment is

$$MLL(S) \simeq n\tfrac{3L+\alpha}{2}\ln(\alpha L) - n\ln(\Gamma(\alpha)) - nL\ln(|\Sigma|) + \tfrac{\alpha-3L}{2}\sum_{k\in S}\ln\left(tr\left(\Sigma^{-1}Z_k\right)\right) + \sum_{k\in S}K_{3L-\alpha}\left\{2\sqrt{\alpha L\, tr\left(\Sigma^{-1}Z_k\right)}\right\}$$

Best α and Σ → Iteration (gradient descent)

Approximation

Σ = segment covariance matrix

$\alpha = 1/(CV_R)^2$ → Method of Moments

DECOMPOSITION INTO TEXTURE AND SPECKLE

Estimate the texture for each channel $\mu = (\mu_{hh}, \mu_{vv}, \mu_{hv})$

-- 5x5 window

Calculate the speckle covariance matrix, speckle $= x - \mu$

The pixel probability model is

$$p(x \mid \Sigma_{speckle}, \alpha) = p(x - \mu \mid \Sigma_{speckle})\, p(\mu_{hh} \mid \alpha_{hh})\, p(\mu_{vv} \mid \alpha_{vv})\, p(\mu_{hv} \mid \alpha_{hv})$$

Independent texture channels

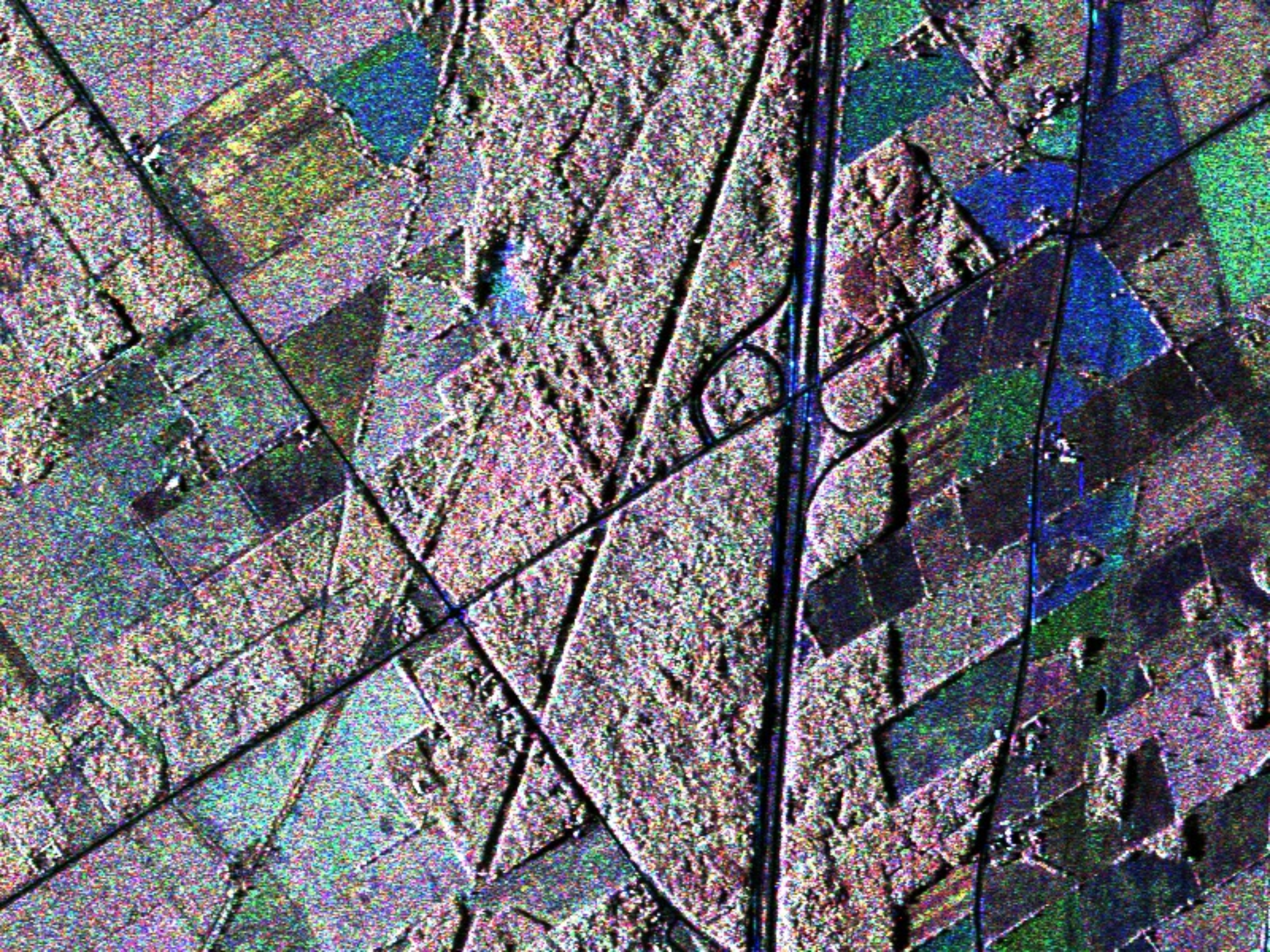
The maximum log likelihood for one segment is

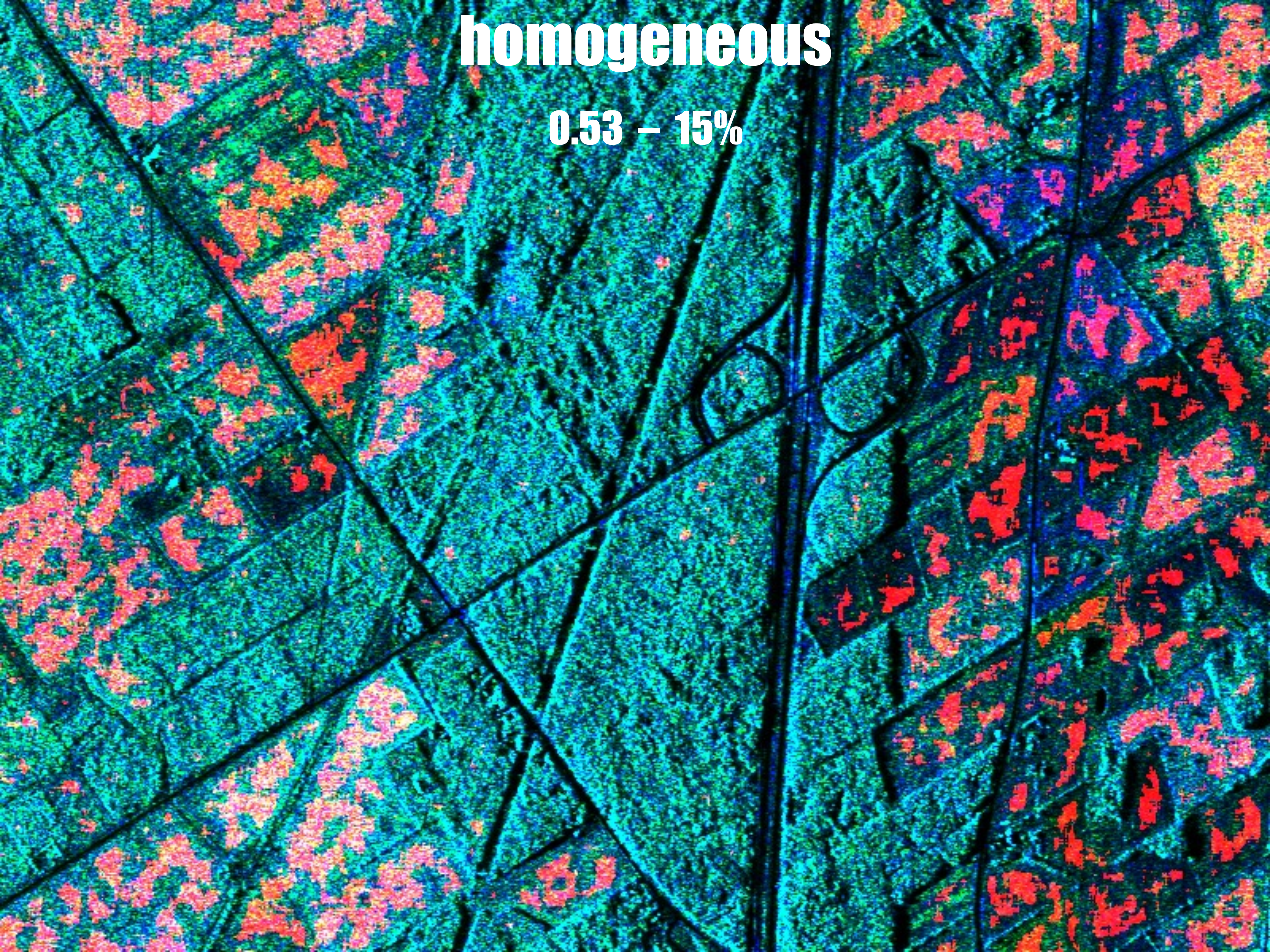
$$MLL = MLL(x-\mu) + MLL(\mu_{hh}) + MLL(\mu_{vv}) + MLL(\mu_{hv})$$

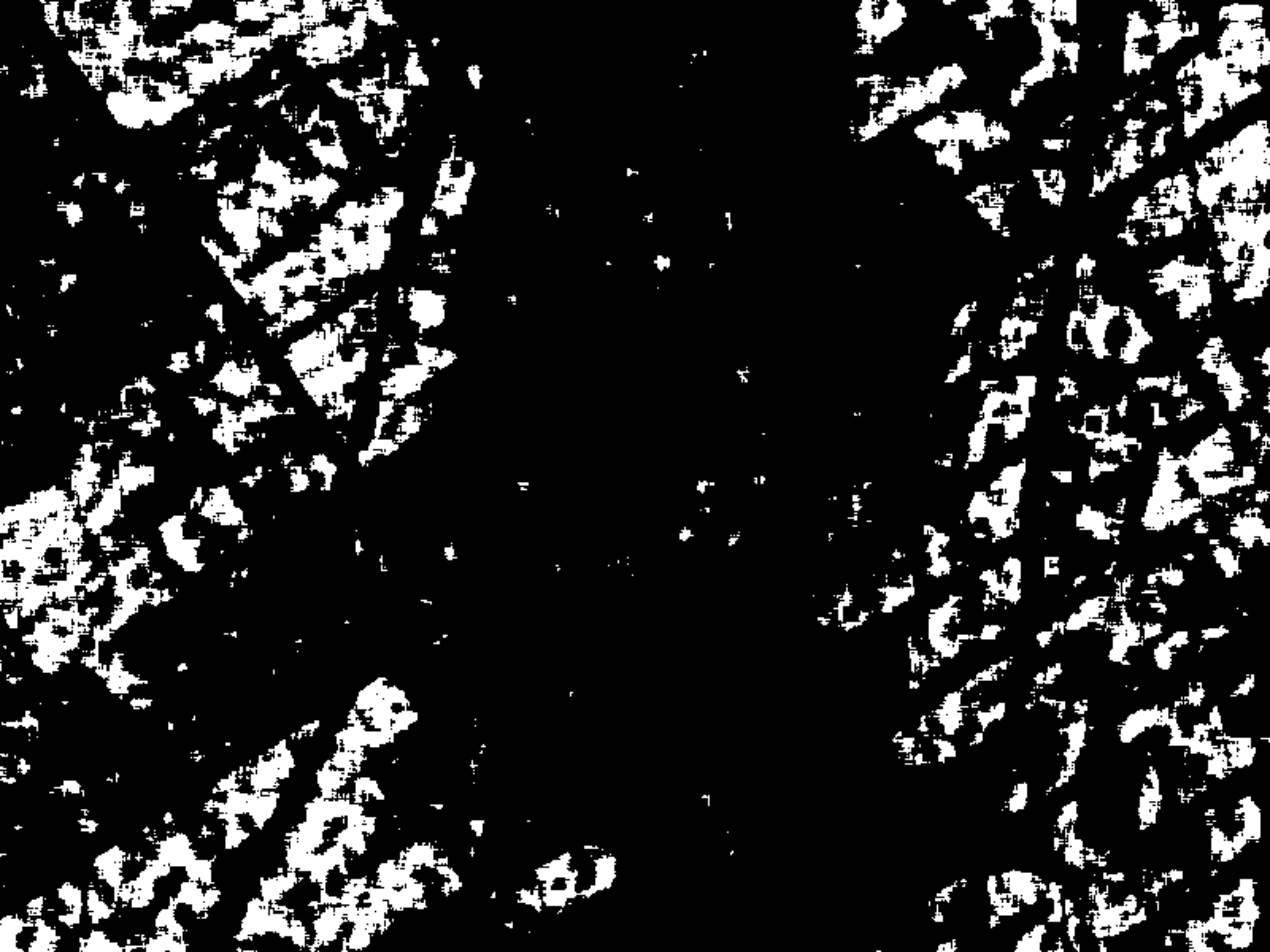
$$MLL(x-\mu) = \sum_{x\in s} \ln\Big(p(x-\mu \mid C_{speckle})\Big) \quad \simeq -n\ln\left|C_{speckle}\right|$$

$$MLL(\mu_{\_}) = \sum_{\mu\in s} \ln\Big(p(\mu_{\_} \mid \alpha_{\_})\Big)$$

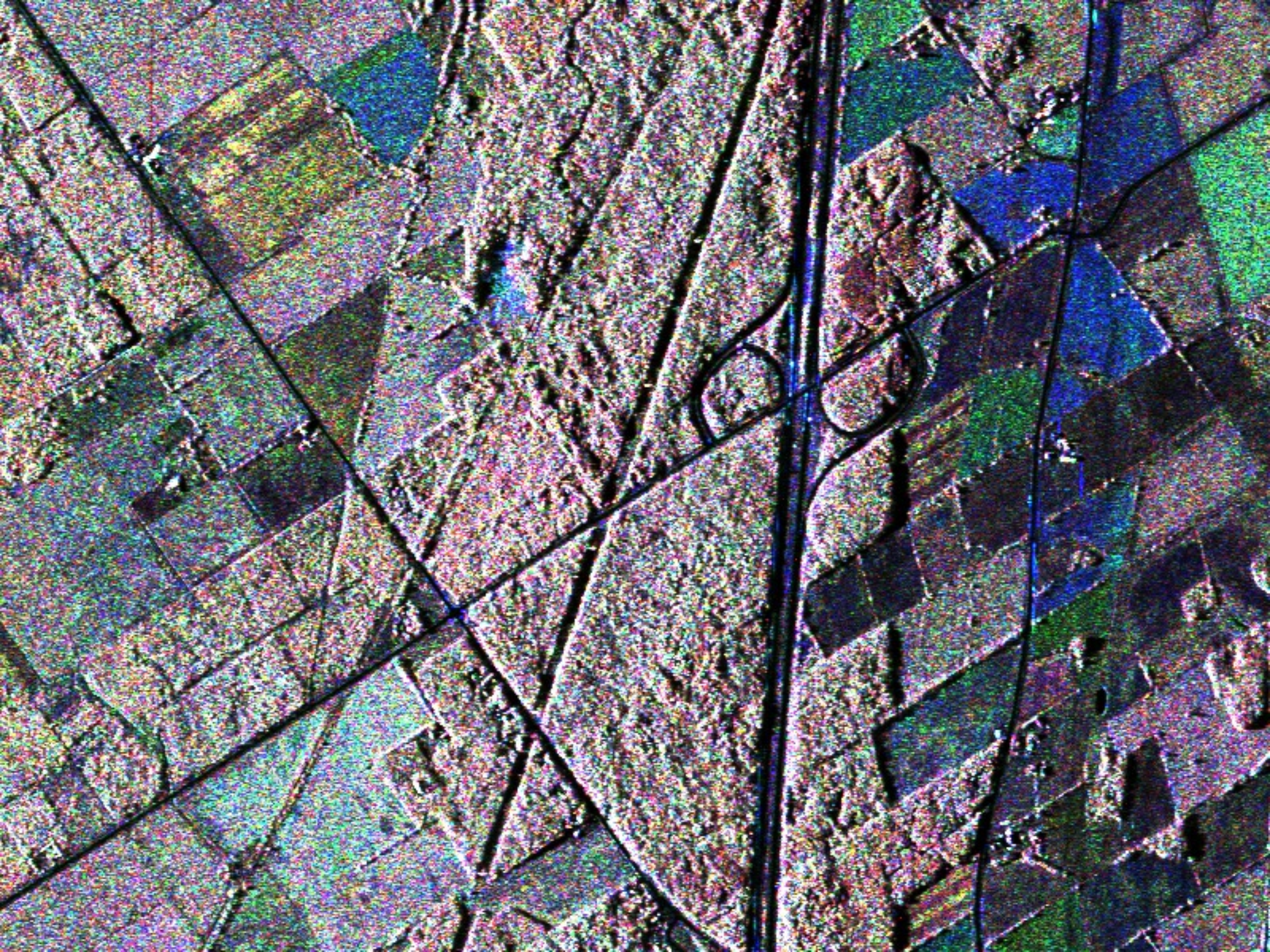
$$\simeq n\Big(\alpha\ln(\alpha) - \ln(\Gamma(\alpha)) - \alpha - \alpha\ln(\bar{\mu})\Big) + (\alpha-1)\sum_{\mu\in s}\ln(\mu)$$

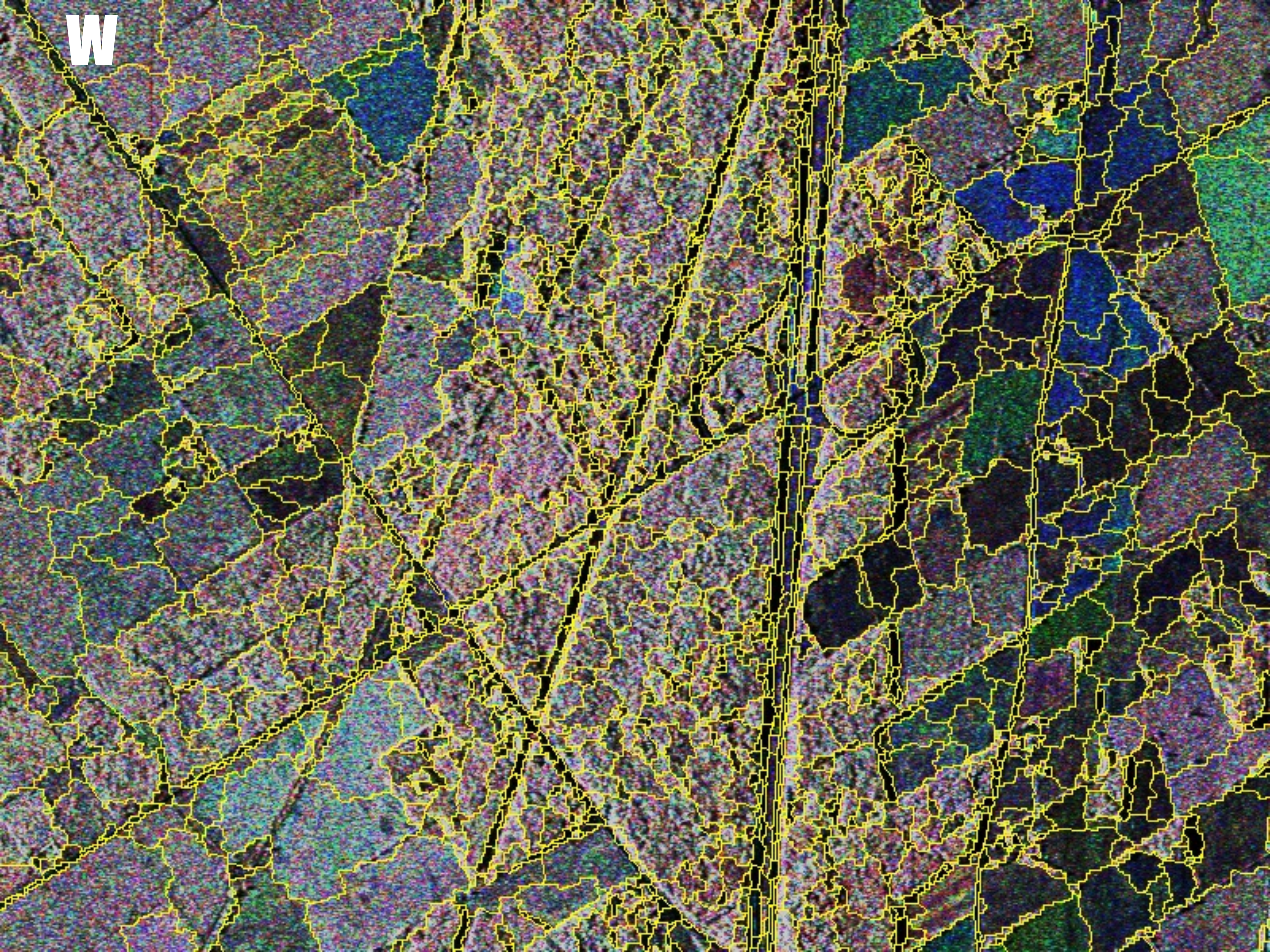



homogeneous
0.53 – 15%

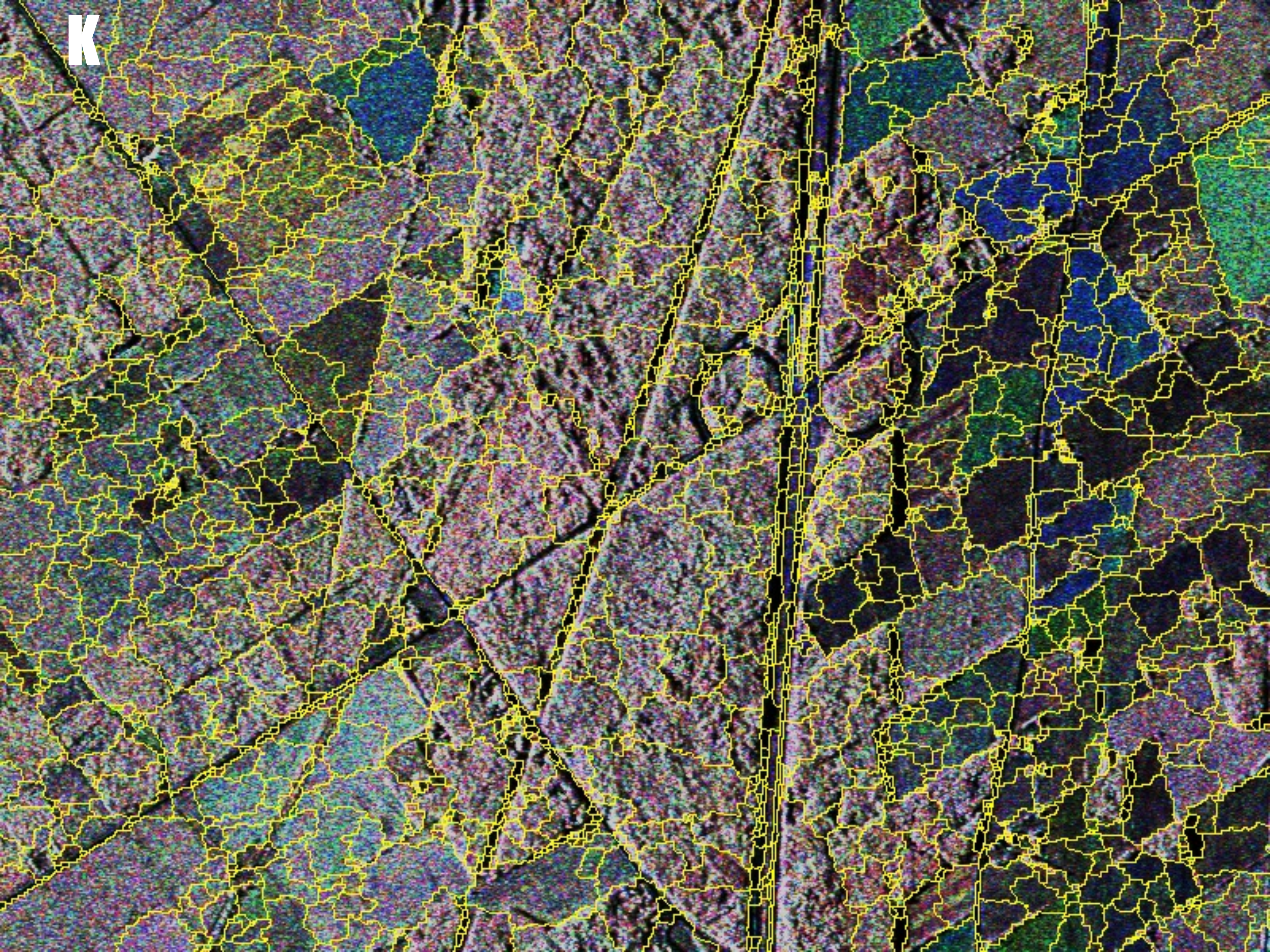


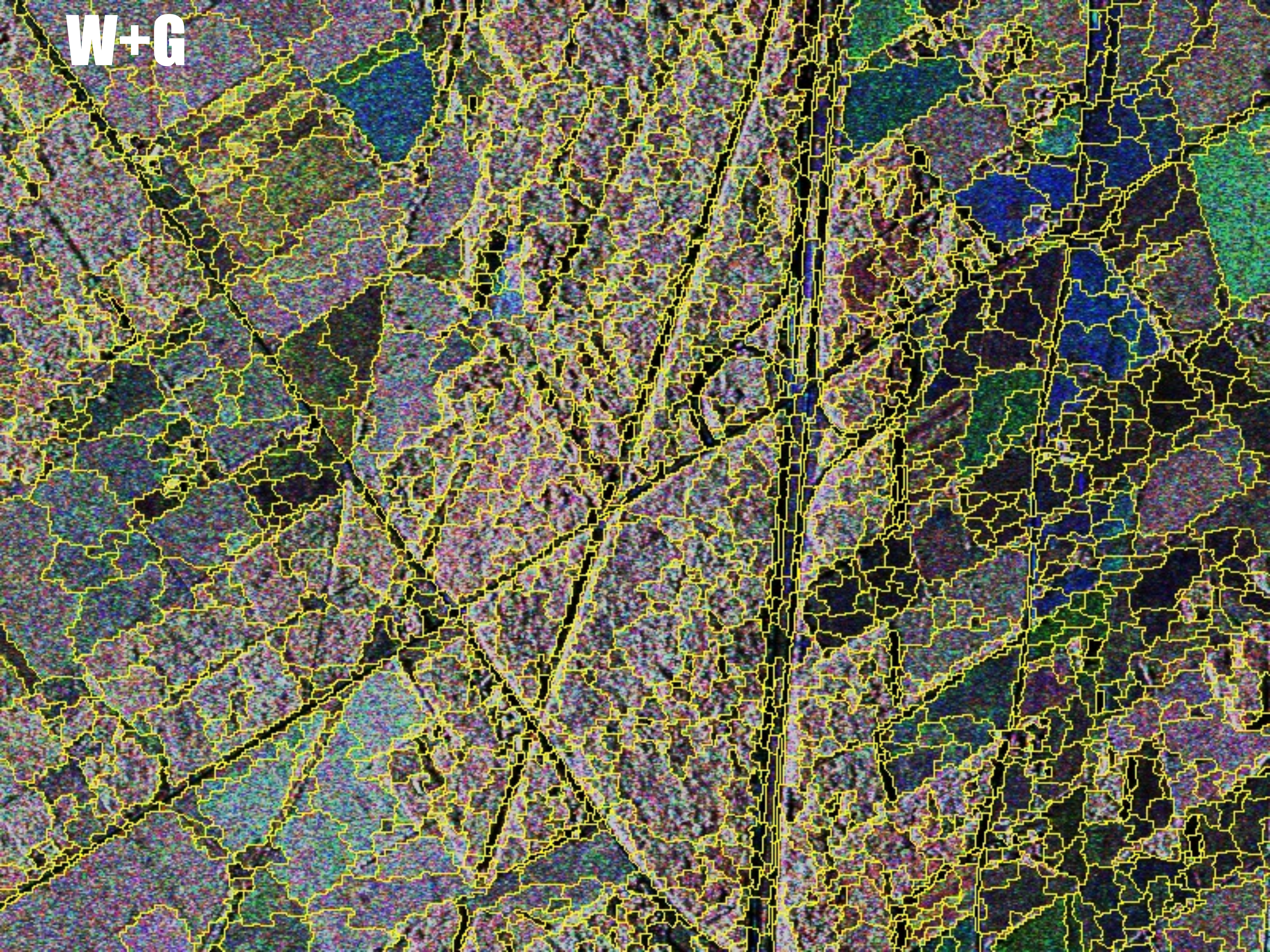

channel difference
0.025 – 22%

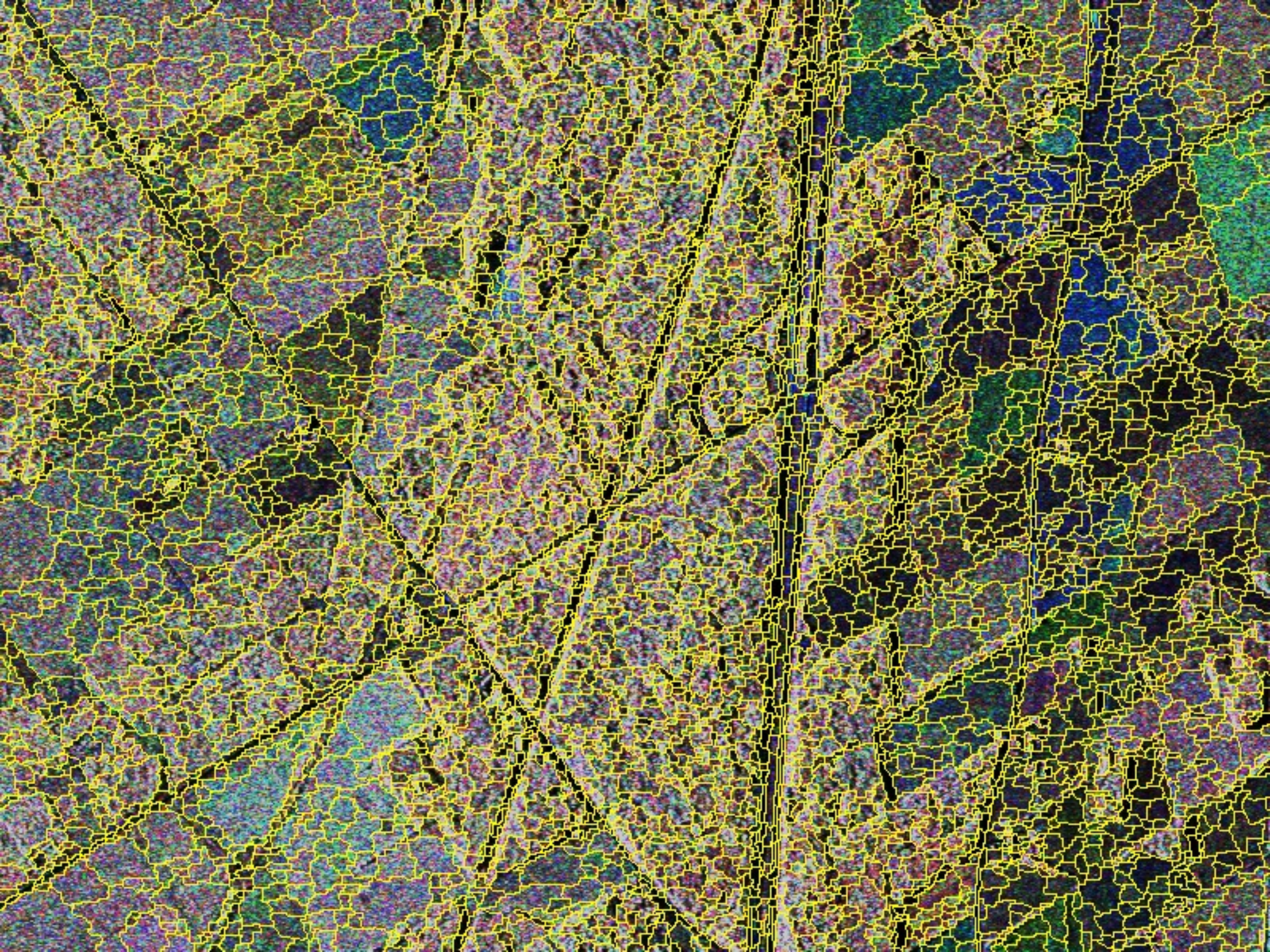


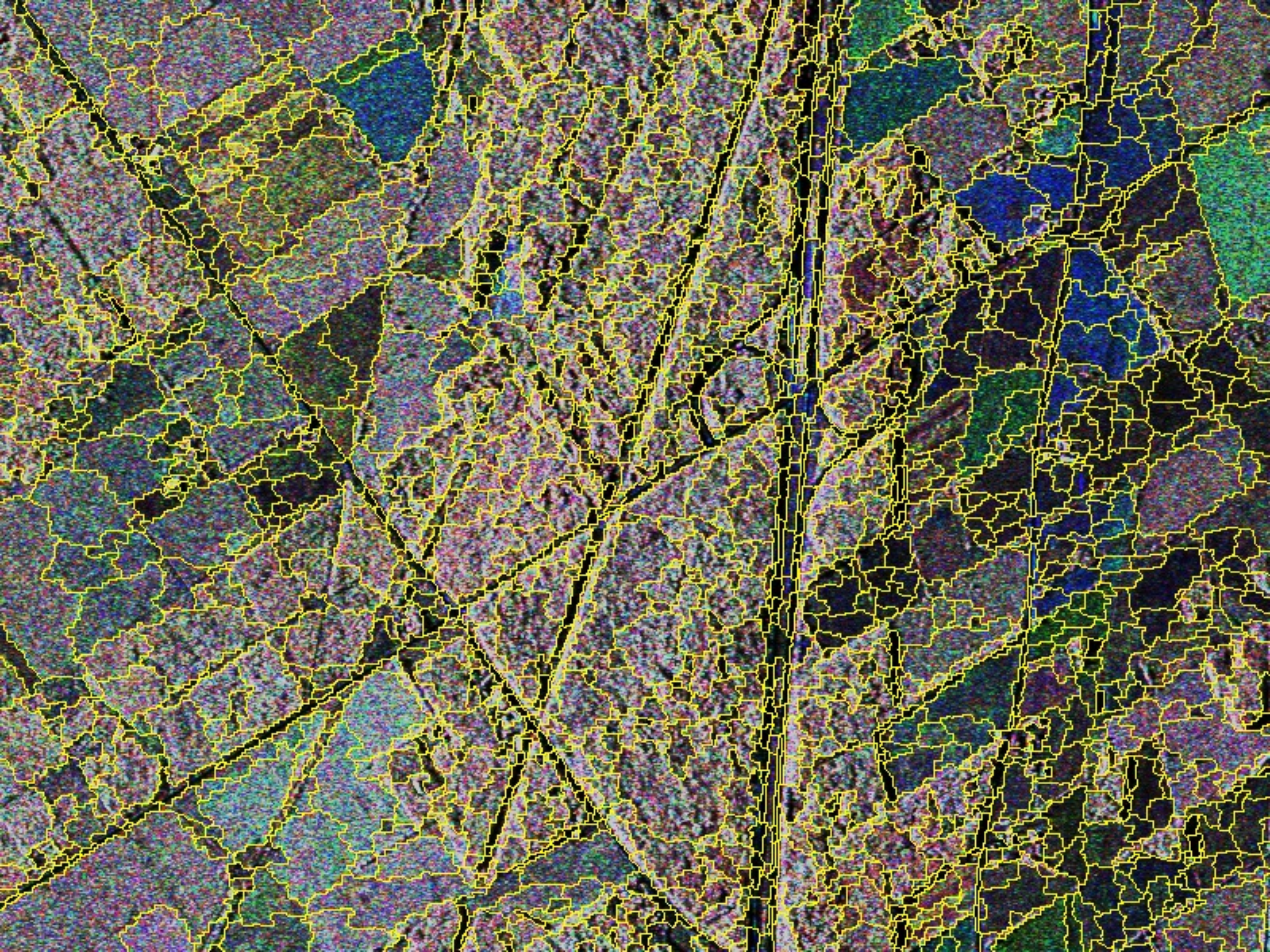

W

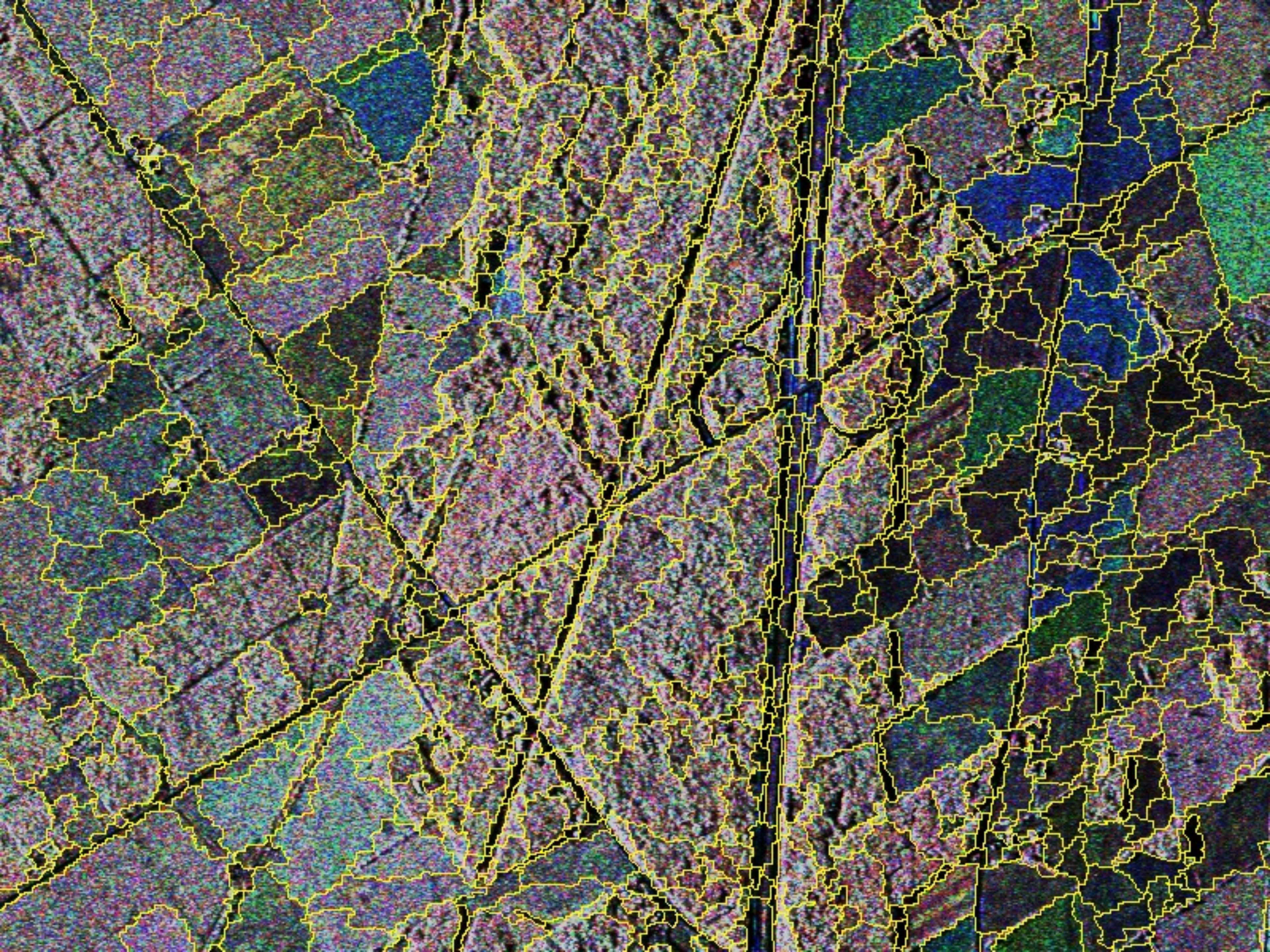
K

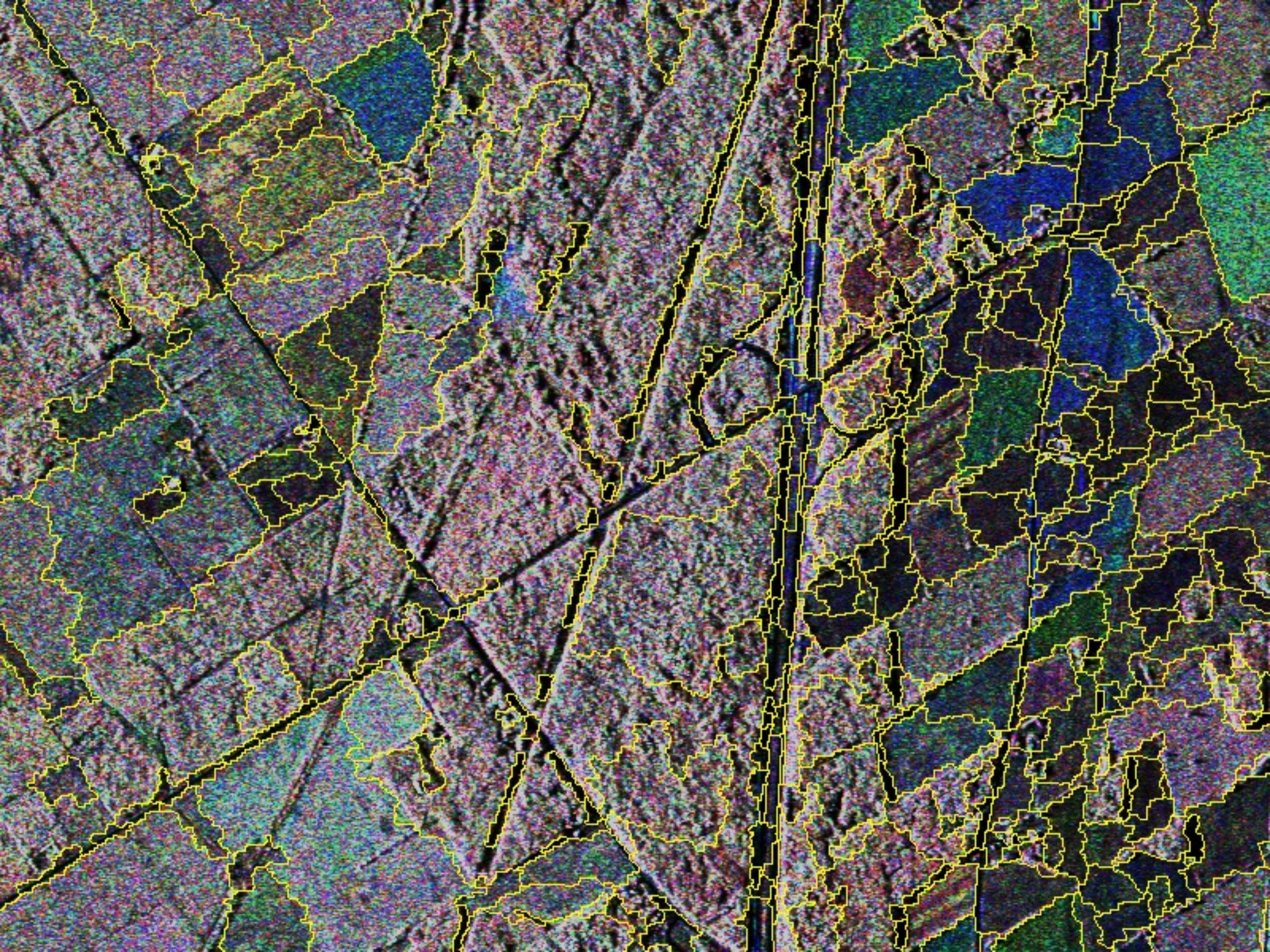












SEGMENT SHAPE CRITERIA

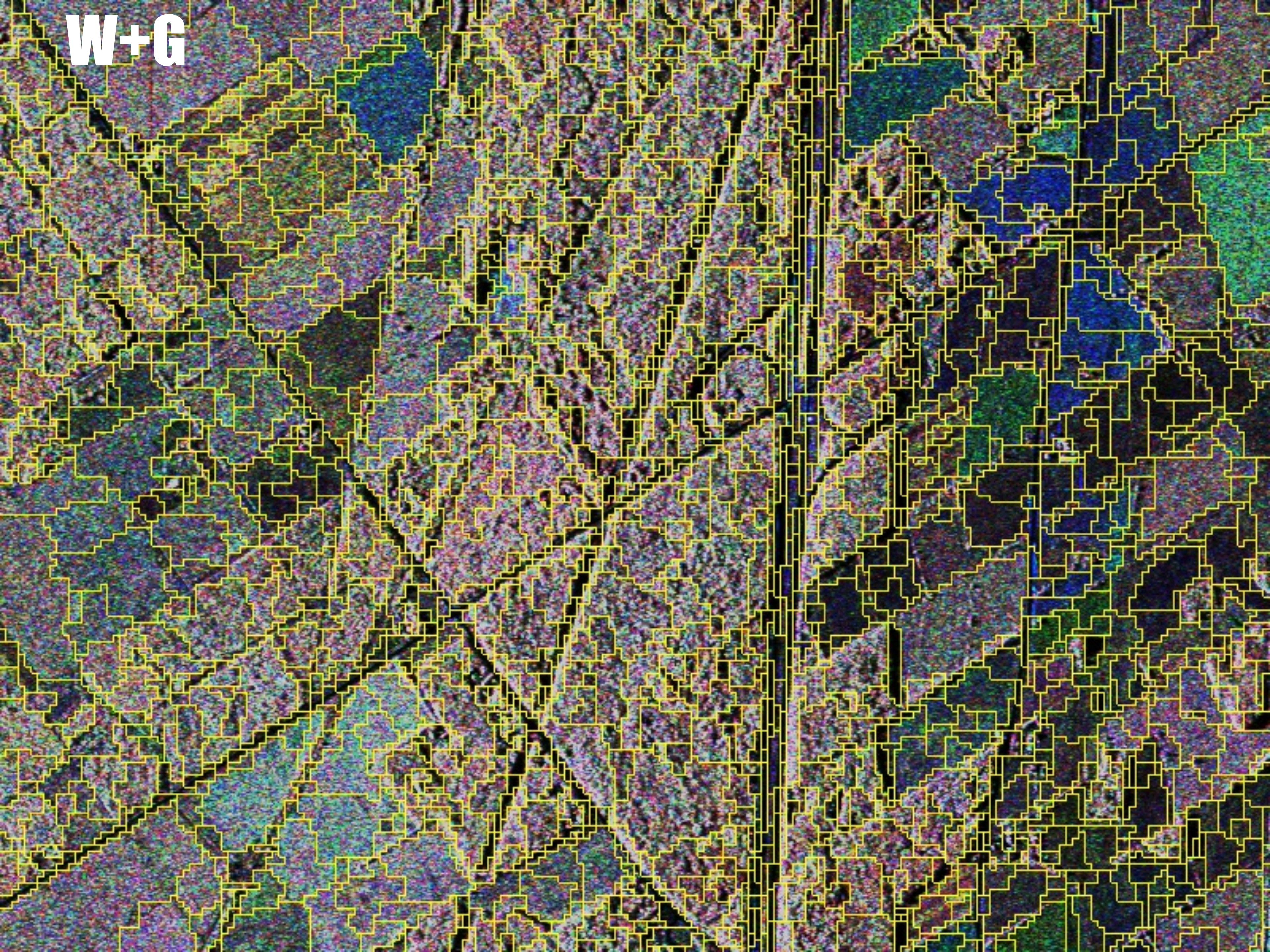
High speckle noise

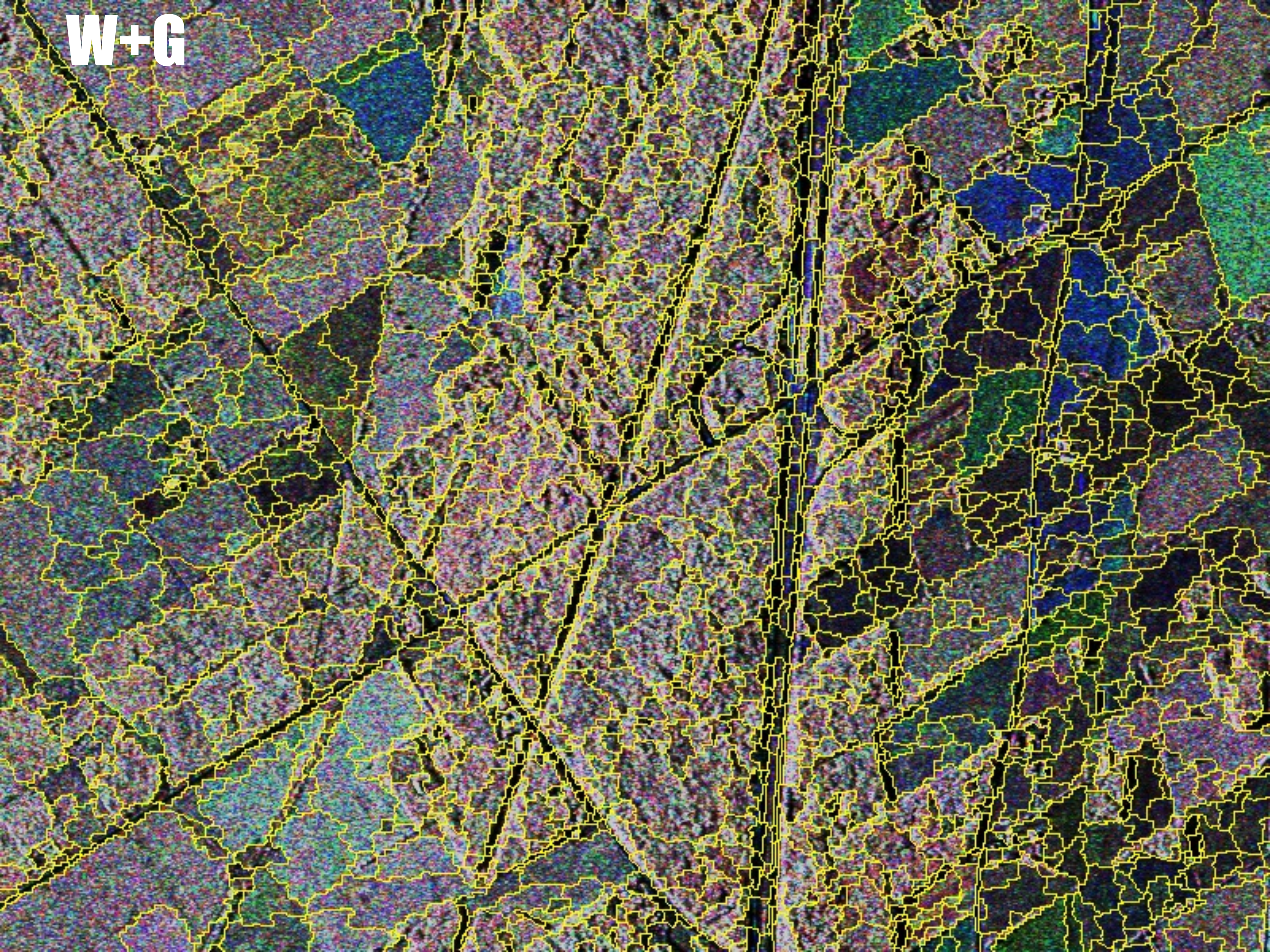
→ first merges produce ill formed segments

- Bonding box – perimeter Cp
- Bonding box – area Ca
- Contour length Cl

New criteria

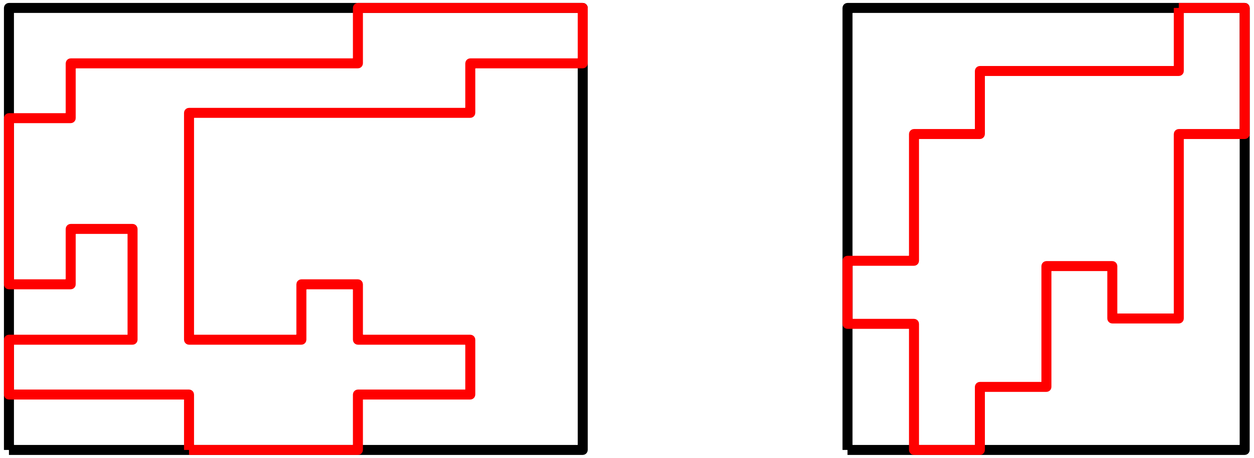
$$C_{i,j}^{contour} = C_{i,j}^{polar} \times Cp^2 \times Ca \times Cl$$


W+G



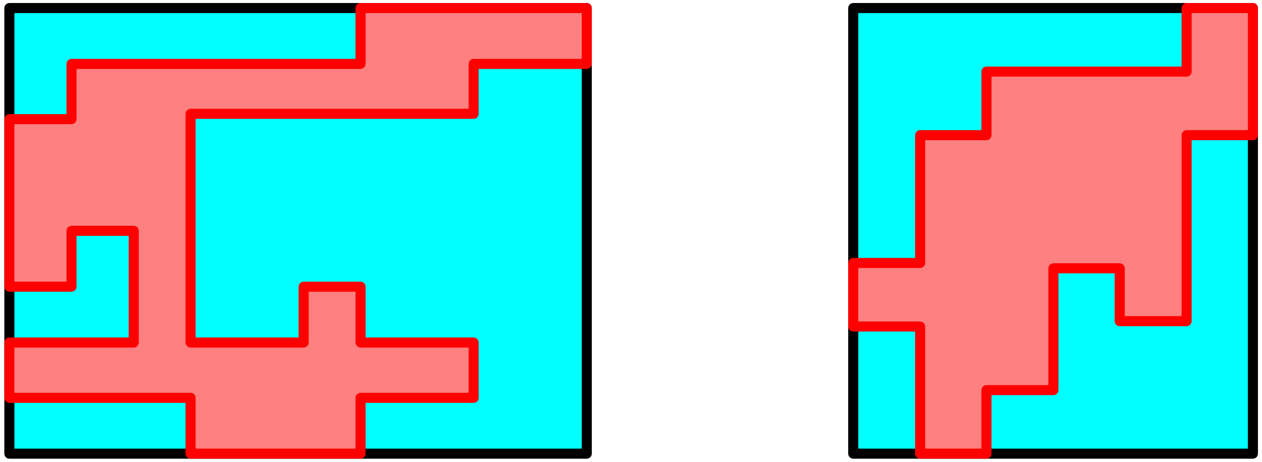
Bonding box – perimeter

$$Cp = \frac{perimeter\ of\ S_i \cup S_j}{perimeter\ of\ bonding\ box}$$



Bonding box – area

$$Ca = \frac{area\ of\ bonding\ box}{area\ of\ S_i \cup S_j}$$

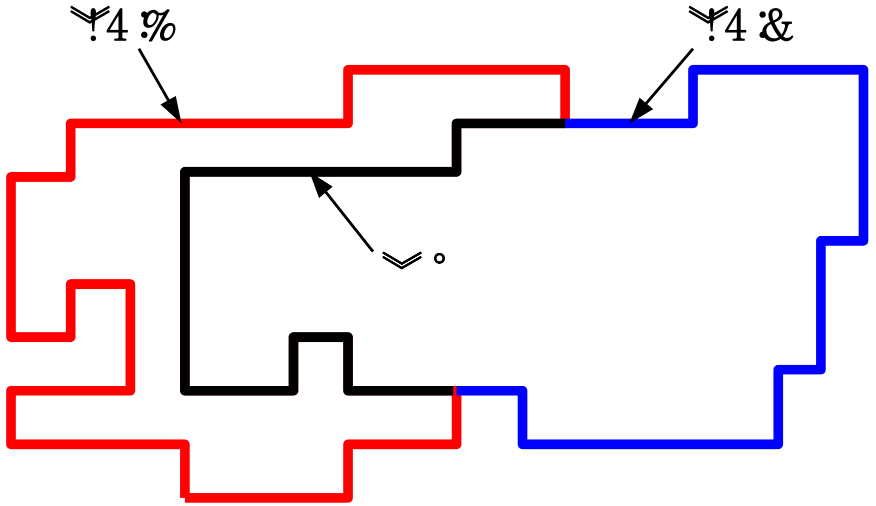


Contour length

$$Lc = \text{length of common part of contours}$$

$$Lex\,i = \text{length of exclusive part for } S_i$$

$$Cl = Min\left\{ \frac{Lex\,i}{Lc}, \quad \frac{Lex\,j}{Lc} \right\}$$



CONCLUSION

- Hierarchical segmentation produces good results
- Good polarimetric criteria for
  homogeneous and textured fields
- Shape criteria are useful

CRITERION FOR SMALL SEGMENTS

The determinant |C| is null for small segments

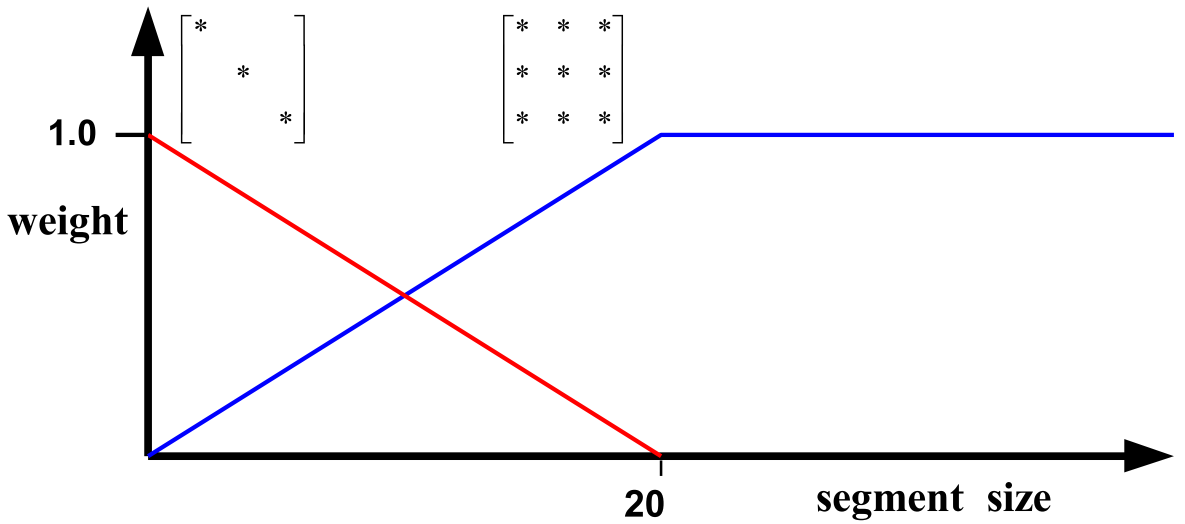
$$C = \frac{1}{n}\begin{bmatrix} \sum hh\,hh^* & \sum hh\,hv^* & \sum hh\,vv^* \\ \sum hv\,hh^* & \sum hv\,hv^* & \sum hv\,vv^* \\ \sum vv\,hh^* & \sum vv\,hv^* & \sum vv\,vv^* \end{bmatrix}$$

Reduce covariance matrix model for small segments

$$\frac{1}{n}\begin{bmatrix} \sum hh\,hh^* & 0 & \sum hh\,vv^* \\ 0 & \sum hv\,hv^* & 0 \\ \sum vv\,hh^* & 0 & \sum vv\,vv^* \end{bmatrix}$$

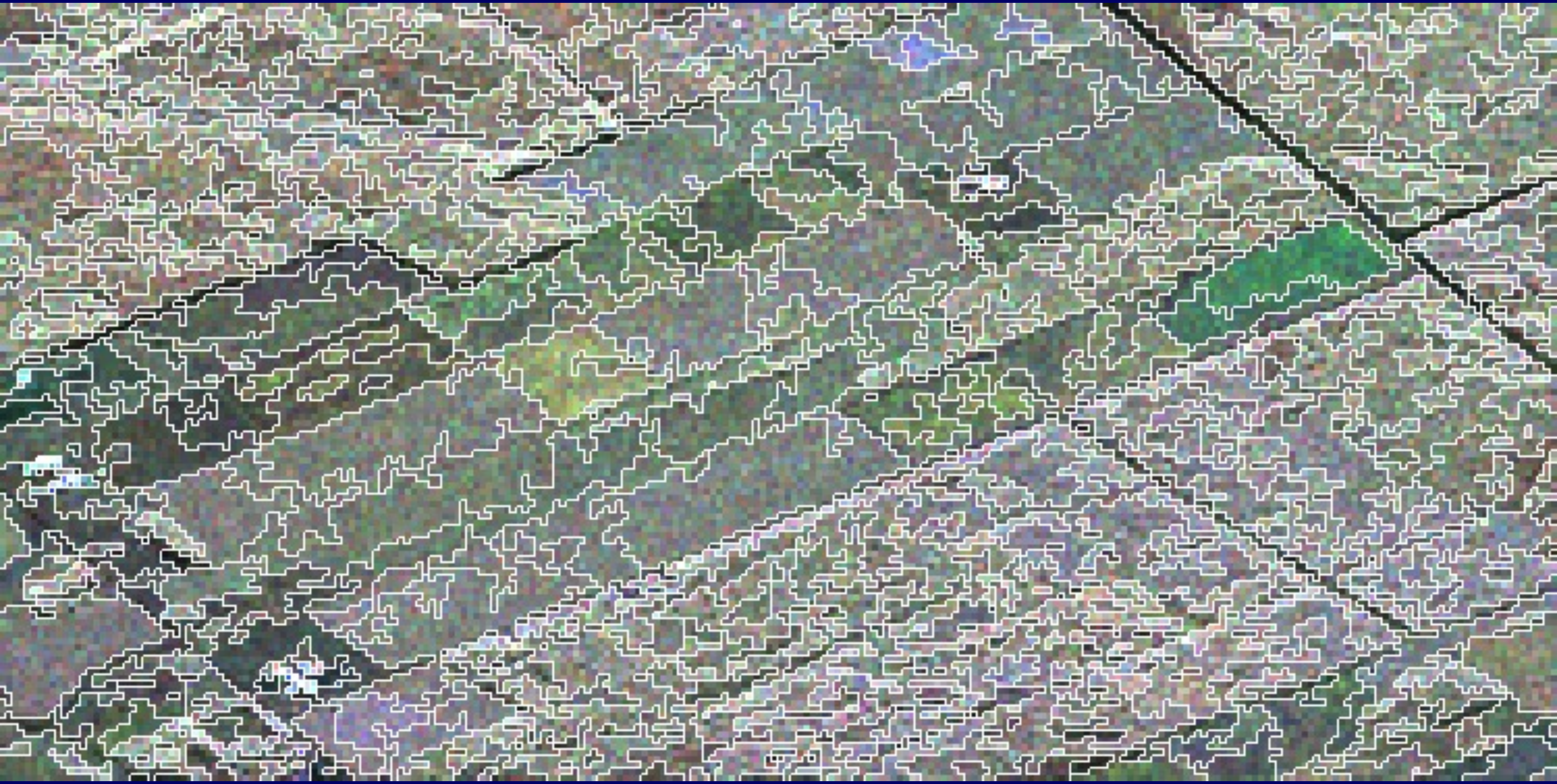
$$\frac{1}{n}\begin{bmatrix} \sum hh\,hh^* & 0 & 0 \\ 0 & \sum hv\,hv^* & 0 \\ 0 & 0 & \sum vv\,vv^* \end{bmatrix}$$

Gradual transition between models

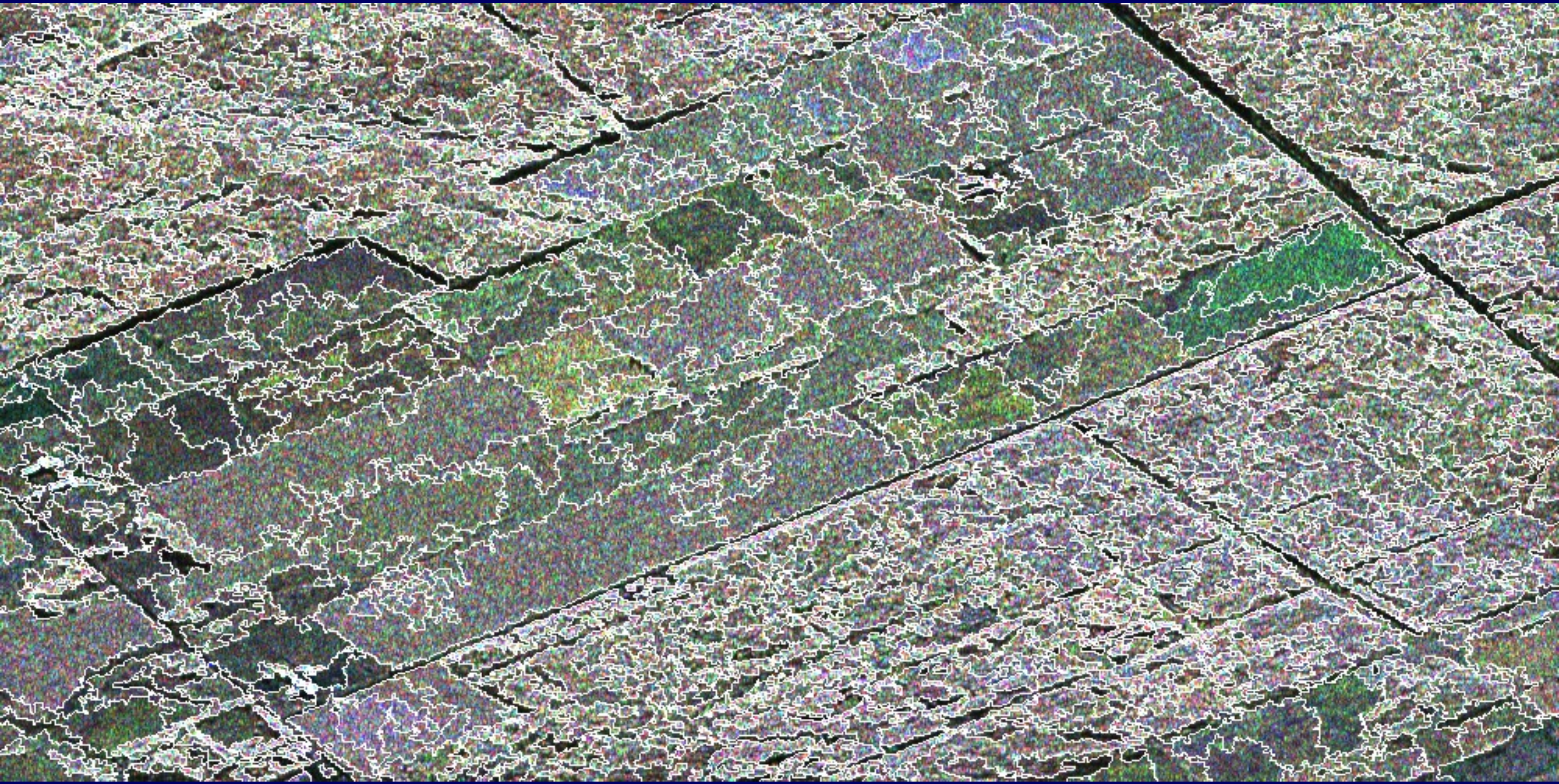



channel difference
0.02 – 15%

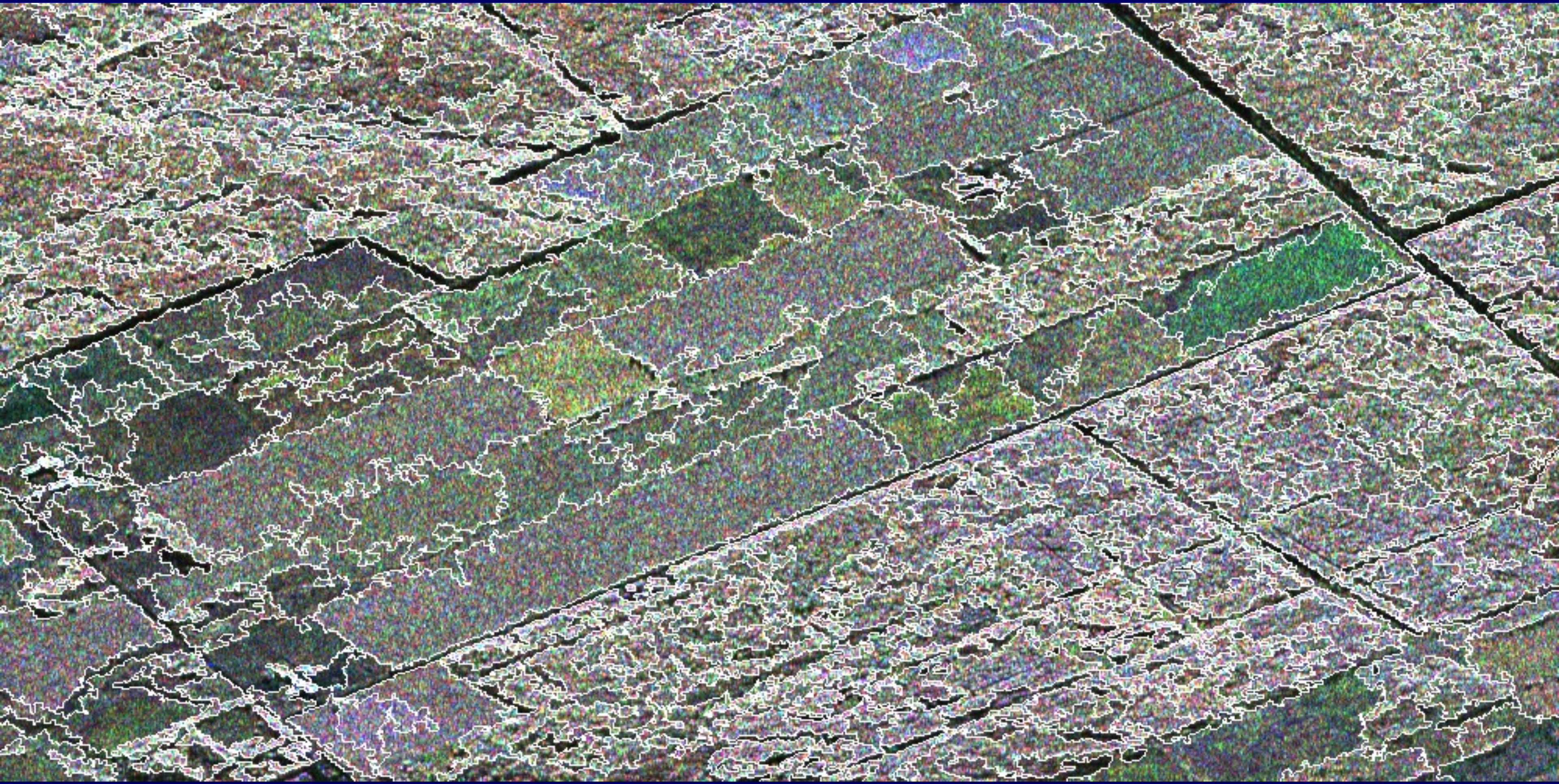
1000 segments – low resolution



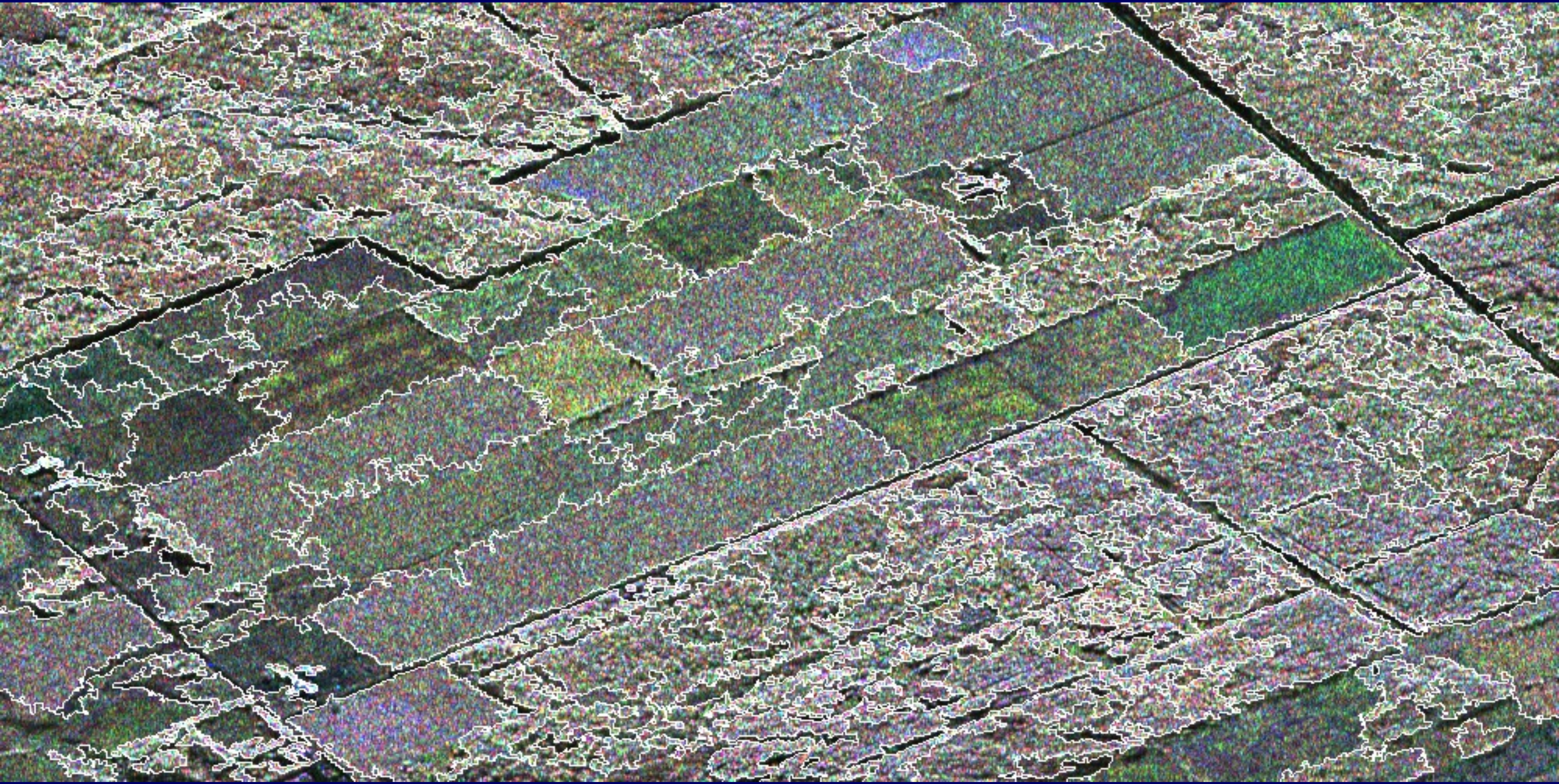
1000 segments



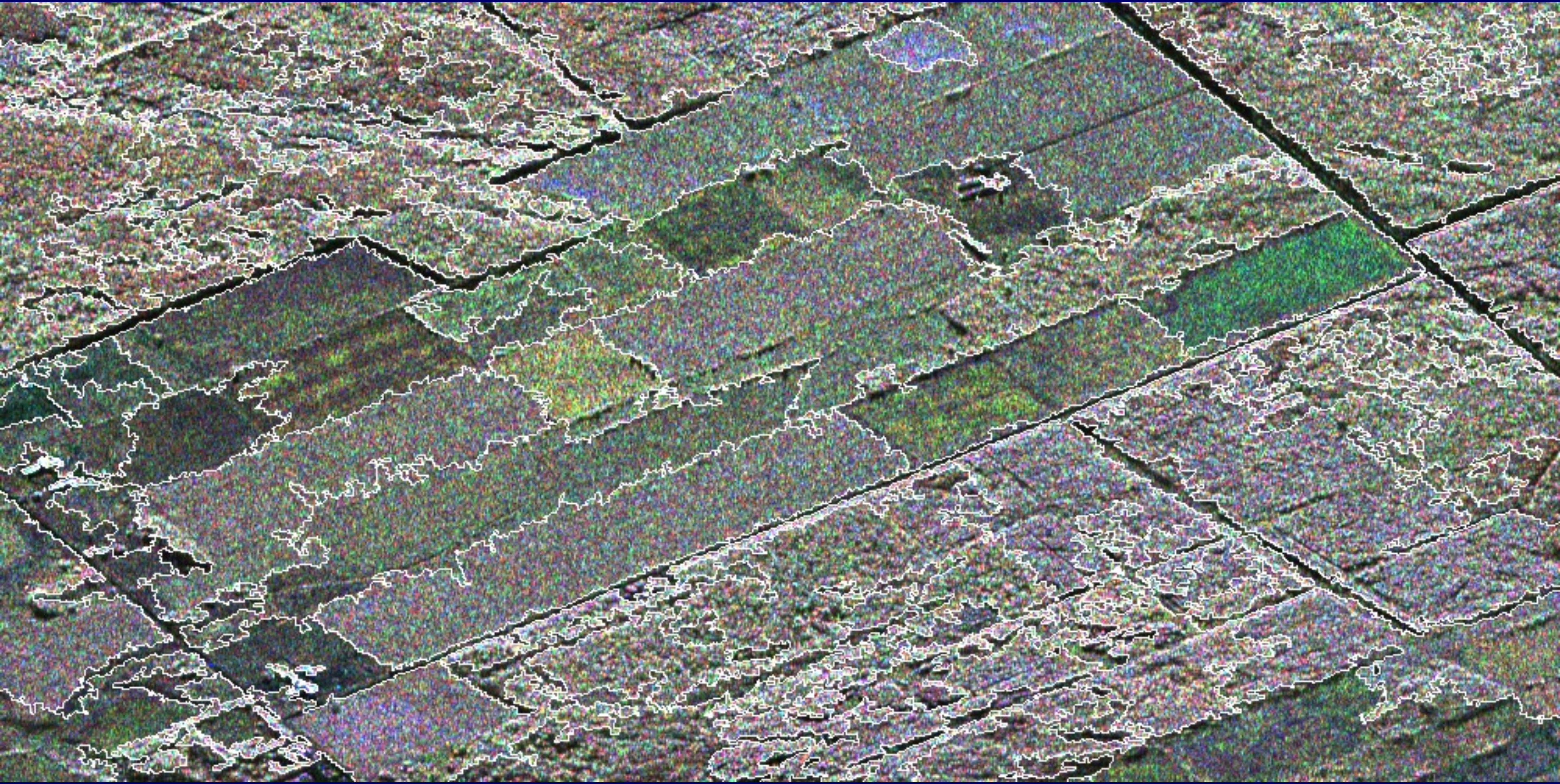
500 segments



200 segments



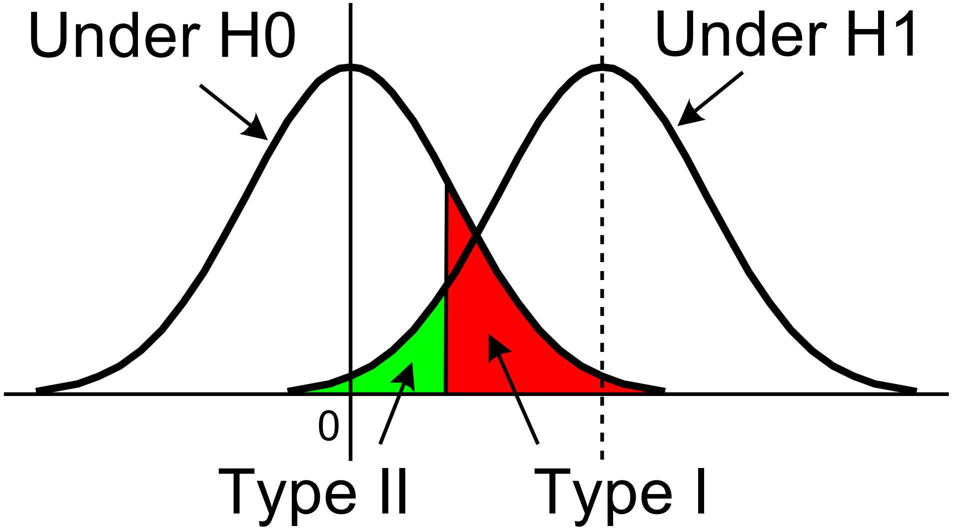
100 segments



Segmentation by hypothesis testing

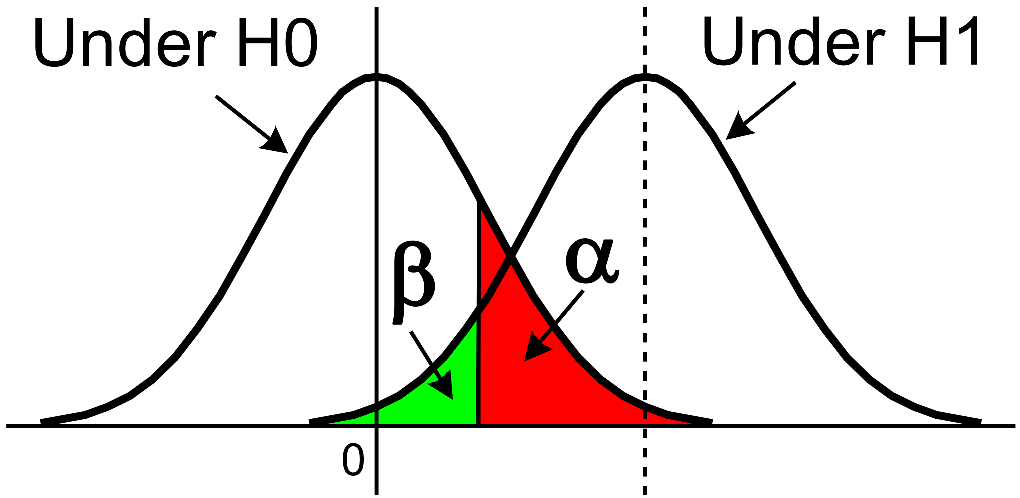
Two hypothesis
H0: segments are similar
H1: segments are different

Distributions of the statistic d under H0 and H1



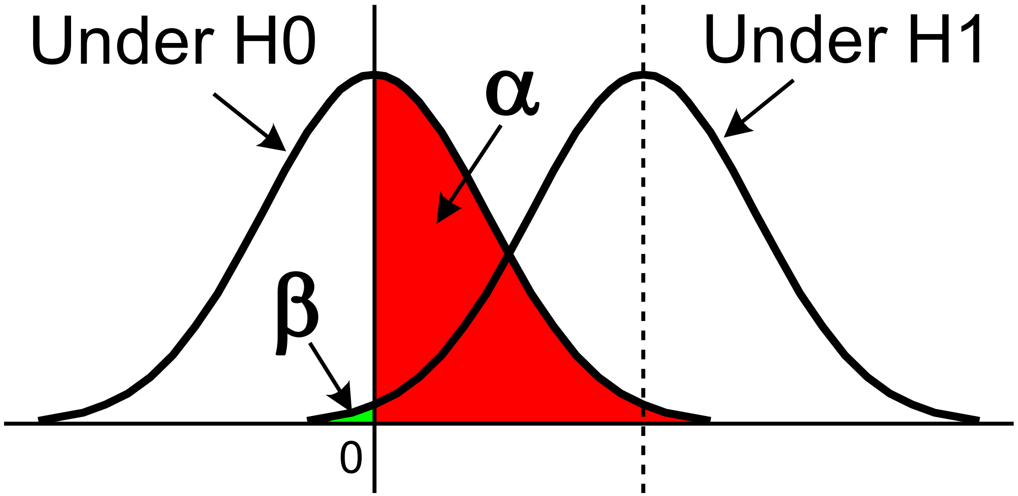
Two types of errors
Type I: not merging similar segments
Type II: merging different segments

α = Prob(Type I errors)
β = Prob(Type II errors)



Select the threshold to minimise α or β, but not both simultaneously

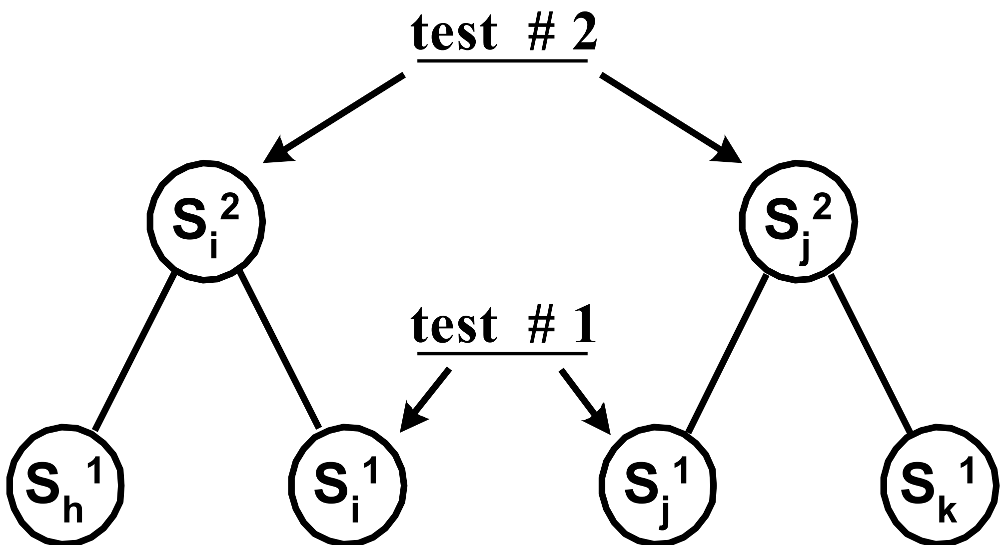
In hierarchical segmentation, type II errors (merging different segments) can not be corrected, while type I errors can be corrected later on.



The distribution of H1 and β are unknown.
Reduce β by increasing α.

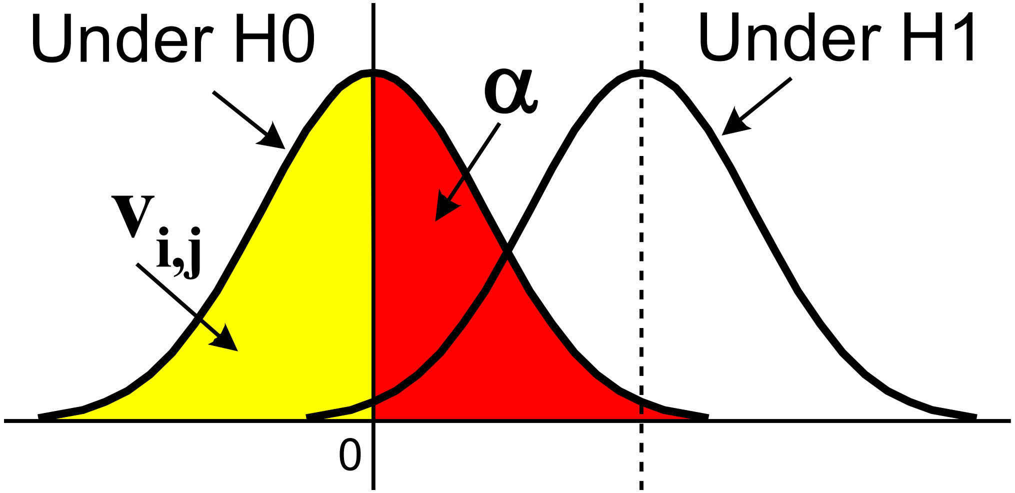
Sequential testing:
α will be reduced as segment sizes increase.

$$\alpha_{1+2+\ldots} \leq \text{minimum}(\ \alpha_1, \alpha_2, \ldots\ )$$
$$\beta_{1+2+\ldots} \geq \text{maximum}(\ \beta_1, \beta_2, \ldots\ )$$



Stepwise criterion

Find and merge the segment pair (i, j) that minimizes $V_{i,j}$ (= 1 - α).



$$V_{i,j} = \mathrm{Prob}(\, d \le d_{i,j} \,;\, H0 \,) \quad (= 1 - \alpha\,).$$