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# **Classification and Segmentation of Radar Polarimetric Images**

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# Classification and Segmentation of Radar Polarimetric Images

- SAR (Synthetic Aperture Radar) images
- Polarimetric SAR images
- Hierarchical Image Segmentation
  - maximum likelihood approximation
- Segmentation of polarimetric images

A Synthetic Aperture Radar (SAR) image of a forested area. The image shows a dense forest with a road or path running through it. A small building or structure is visible on the right side. The text "SAR Images" is overlaid in yellow. The image is in grayscale and has a grainy, speckled appearance characteristic of SAR data.

# SAR Images

# SAR (Synthetic Aperture Radar) IMAGE

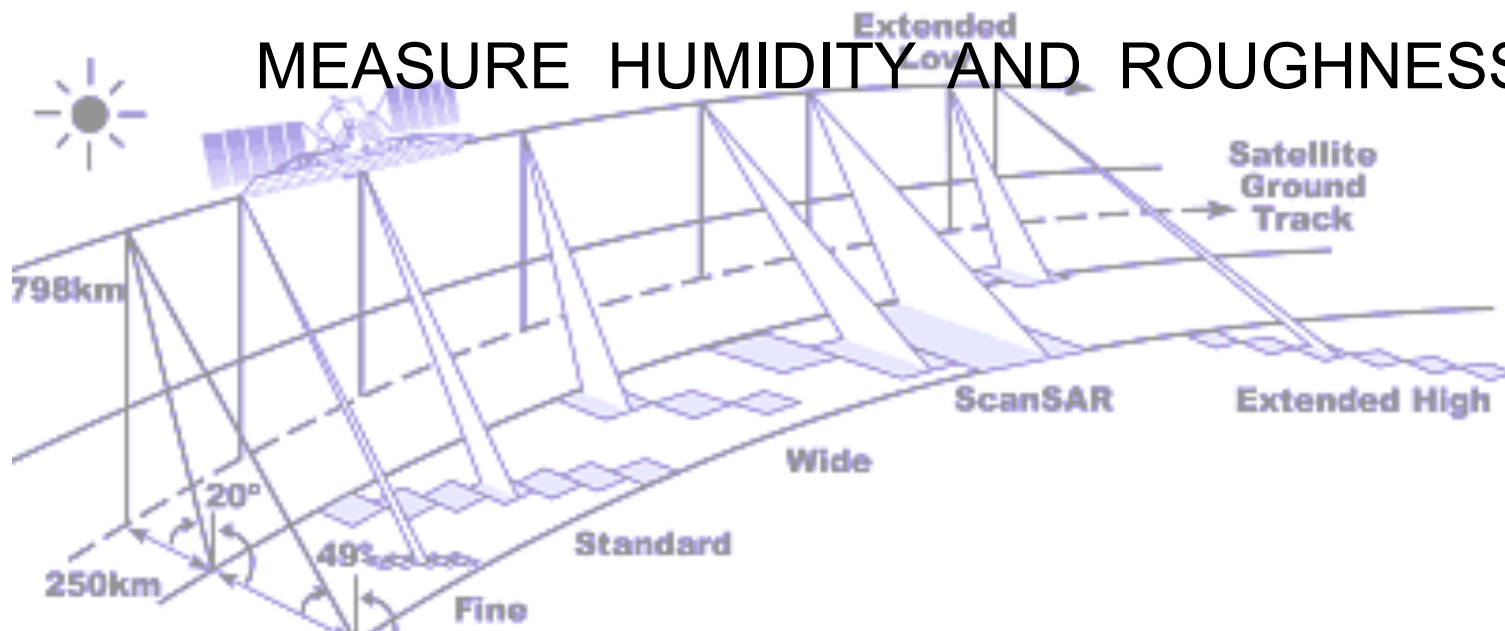
ACTIVE SENSOR → SEND A COHERENT SIGNAL

ONE CHANNEL → GREY SCALE IMAGE

LONG WAVELENGTH → 0.1 TO 1 METER

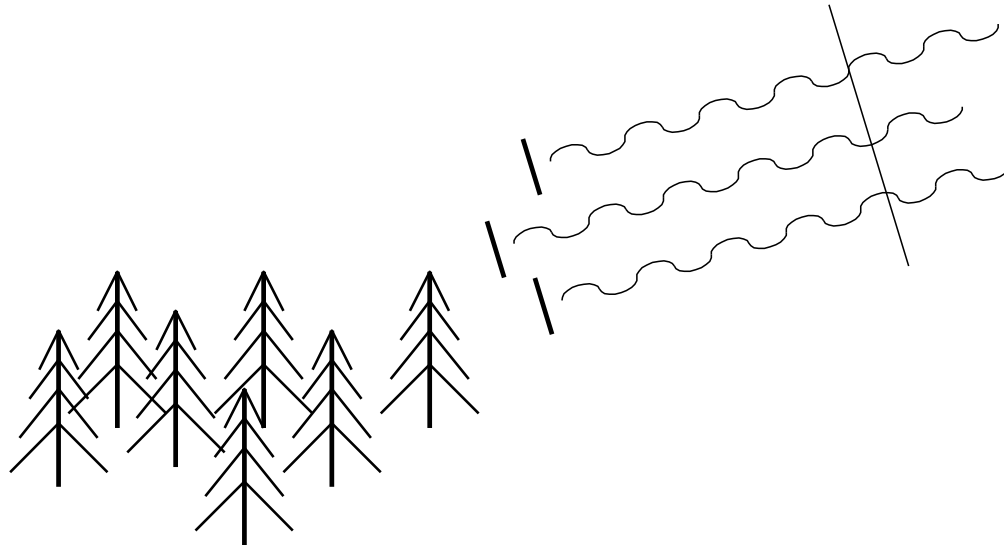
PASS THROUGH CLOUDS

MEASURE HUMIDITY AND ROUGHNESS



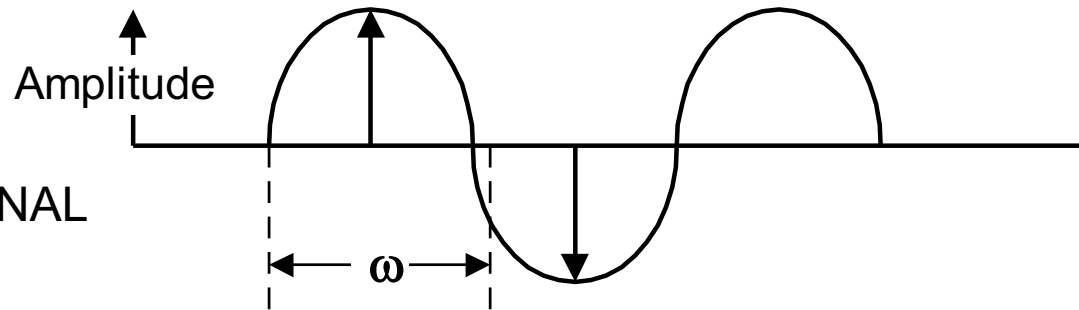
# SAR (Synthetic Aperture Radar) IMAGE

SAR IMAGE → COHERENT SIGNAL (RADAR)  
→ INTERFERENCE PATTERN



RETURNED SIGNAL HAS  
**AMPLITUDE AND PHASE**

THE NOISE IS PROPORTIONAL  
TO THE AMPLITUDE



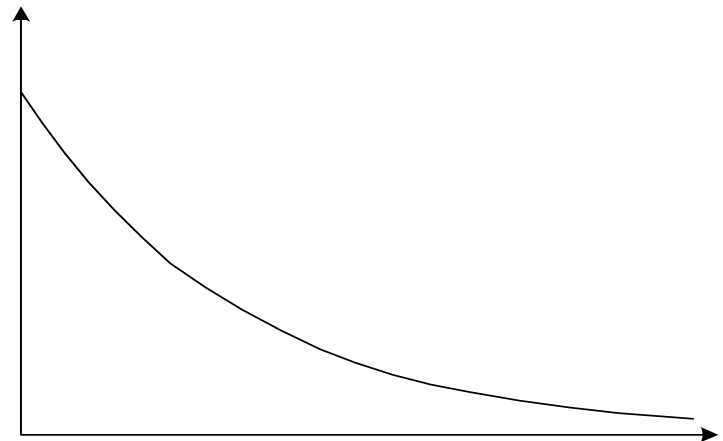
SIGNAL DISTRIBUTION

POWER OR INTENSITY

**EXPONENTIAL DISTRIBUTION**

$$p(I) = \frac{1}{\lambda} \text{Exp} \left\{ \frac{-I}{\lambda} \right\}$$

where  $\sigma_I = E(I) = \lambda$



## MULTI-LOOK IMAGE

L = NUMBER OF LOOKS

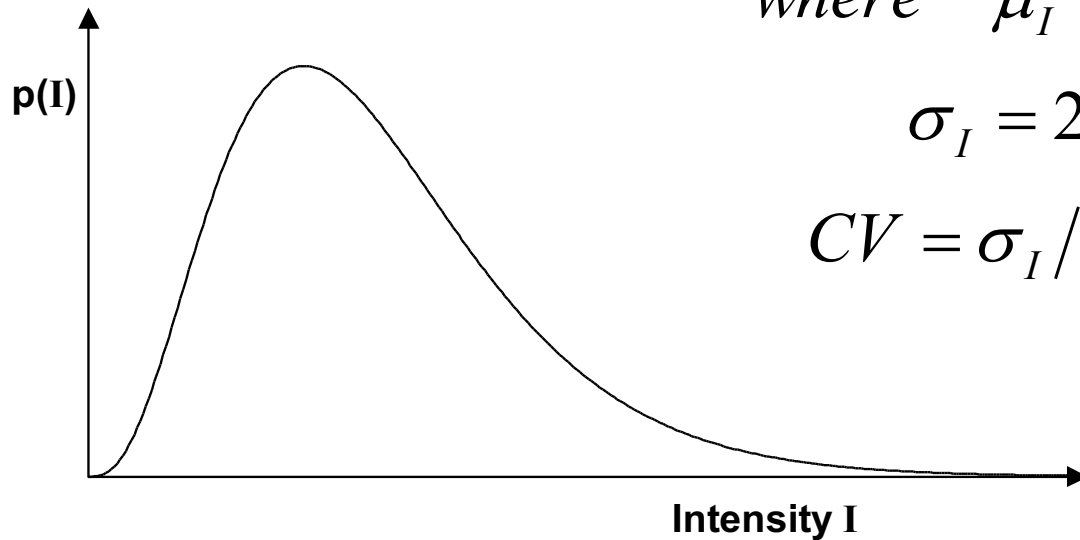
INTENSITY FOLLOWS A **GAMMA** DISTRIBUTION

$$p(I) = \left(\frac{L}{2\sigma^2}\right)^L \frac{I^{L-1}}{\Gamma(L)} \text{Exp}\left\{\frac{-LI}{2\sigma^2}\right\}$$

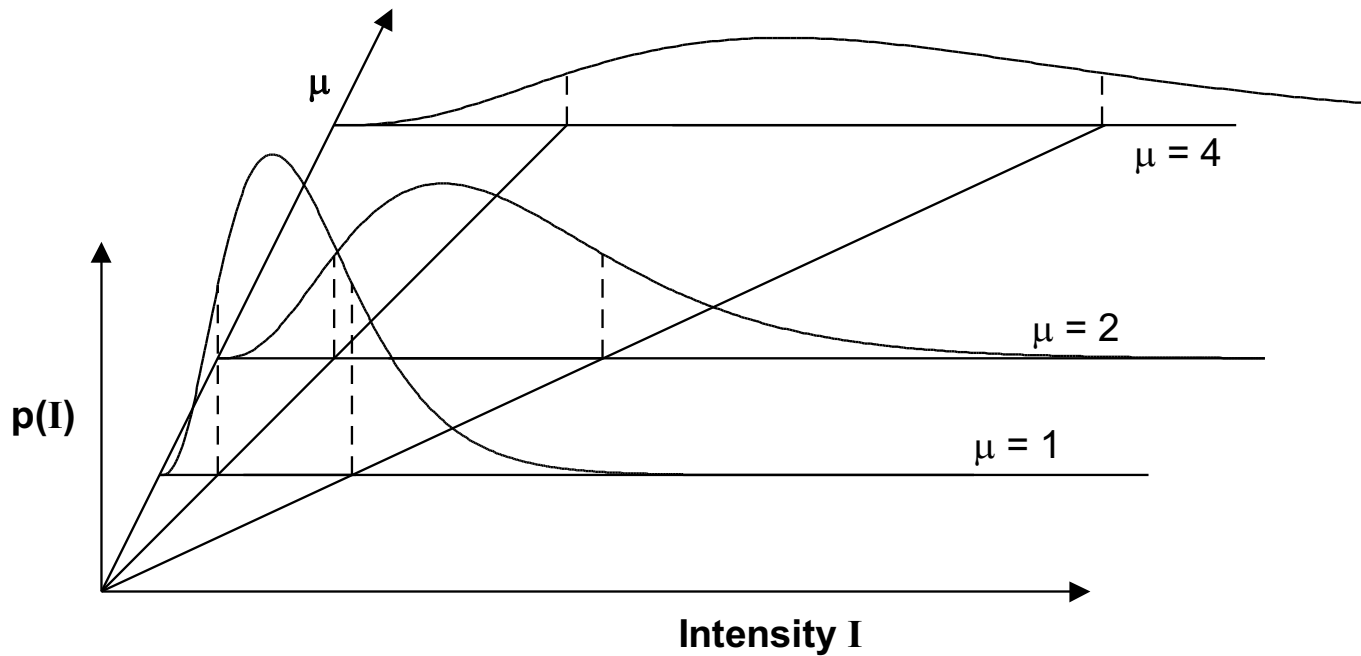
$$\text{where } \mu_I = E(I) = 2\sigma^2$$

$$\sigma_I = 2\sigma^2 / \sqrt{L}$$

$$CV = \sigma_I / \mu_I = 1 / \sqrt{L}$$



# MULTIPLICATIVE NOISE



NOISE IS PROPORTIONAL TO INTENSITY



**1000x1000 SAR image**

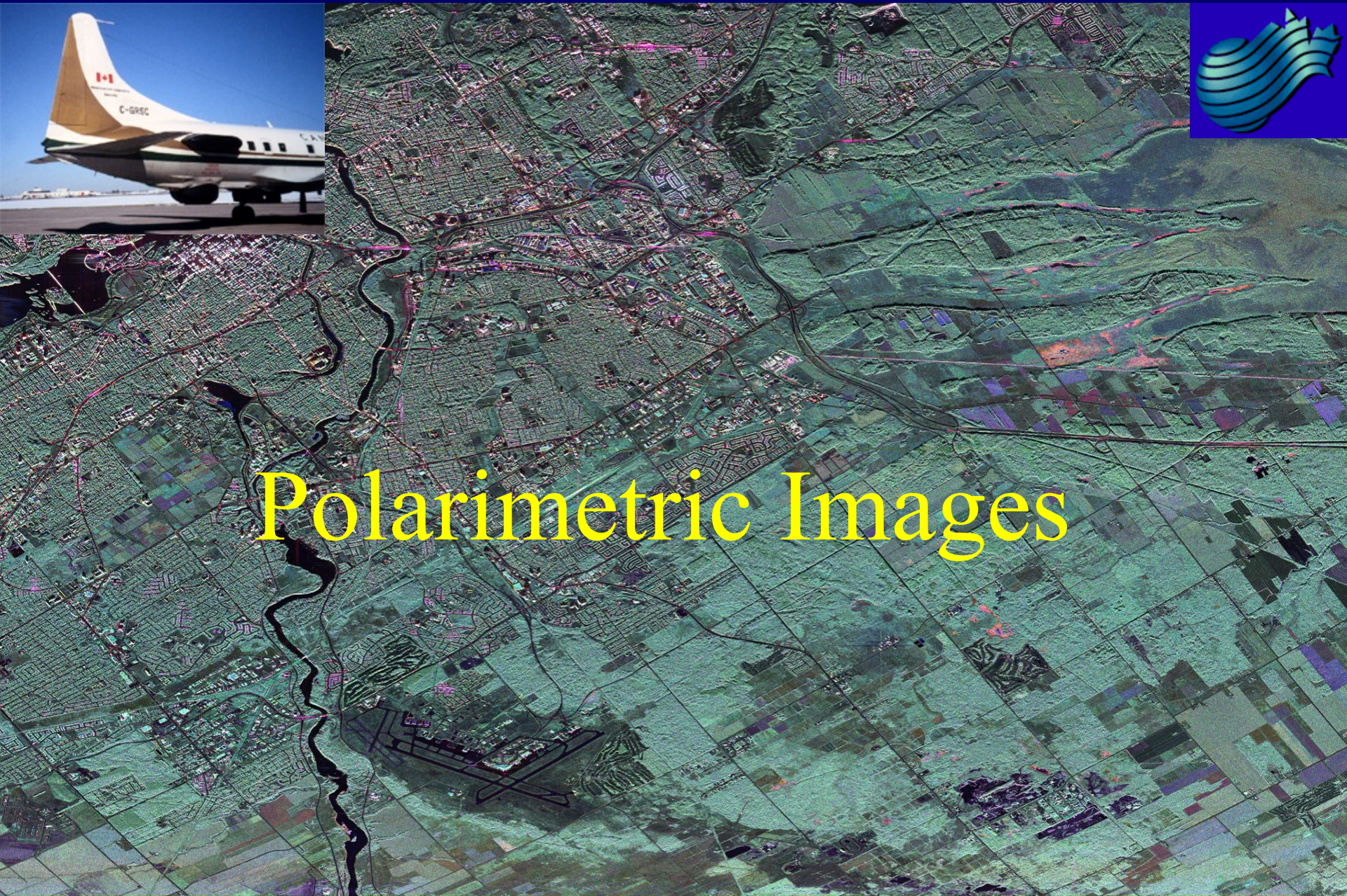


**SAR image**



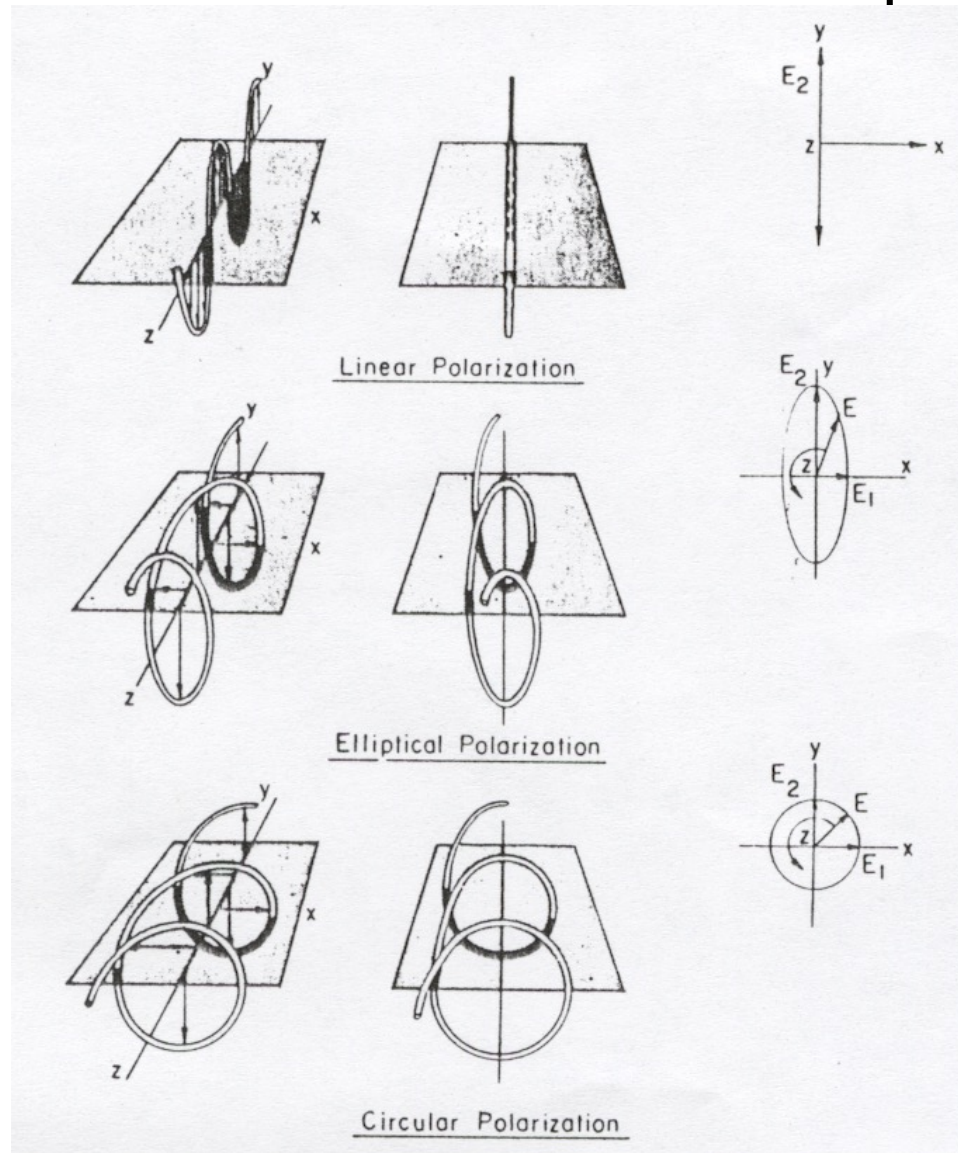


# Polarimetric Images





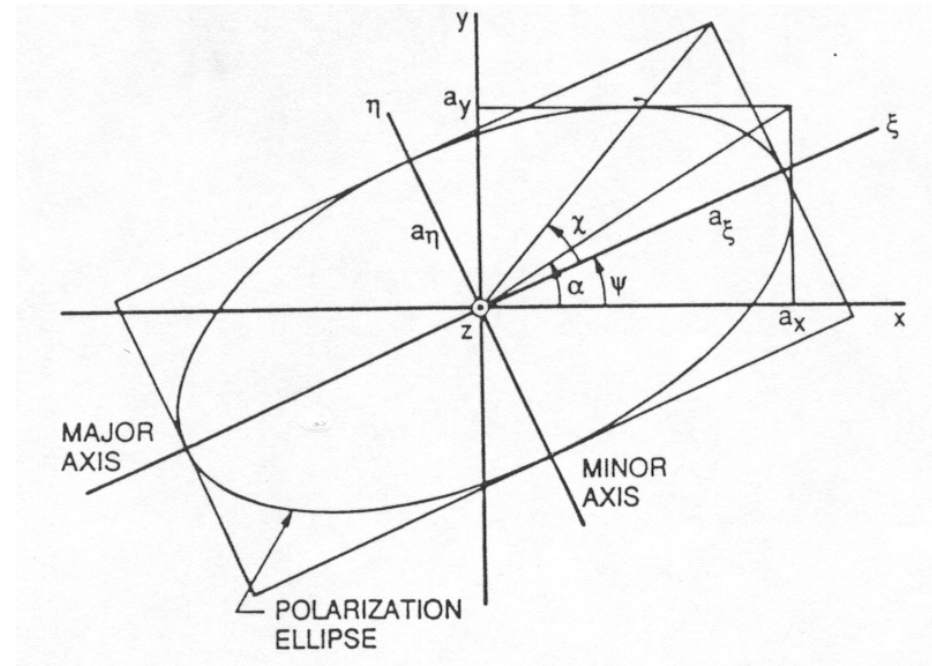
# Polarization of an e.m. monochromatic plane wave





## Polarization of an e.m. monochromatic plane wave

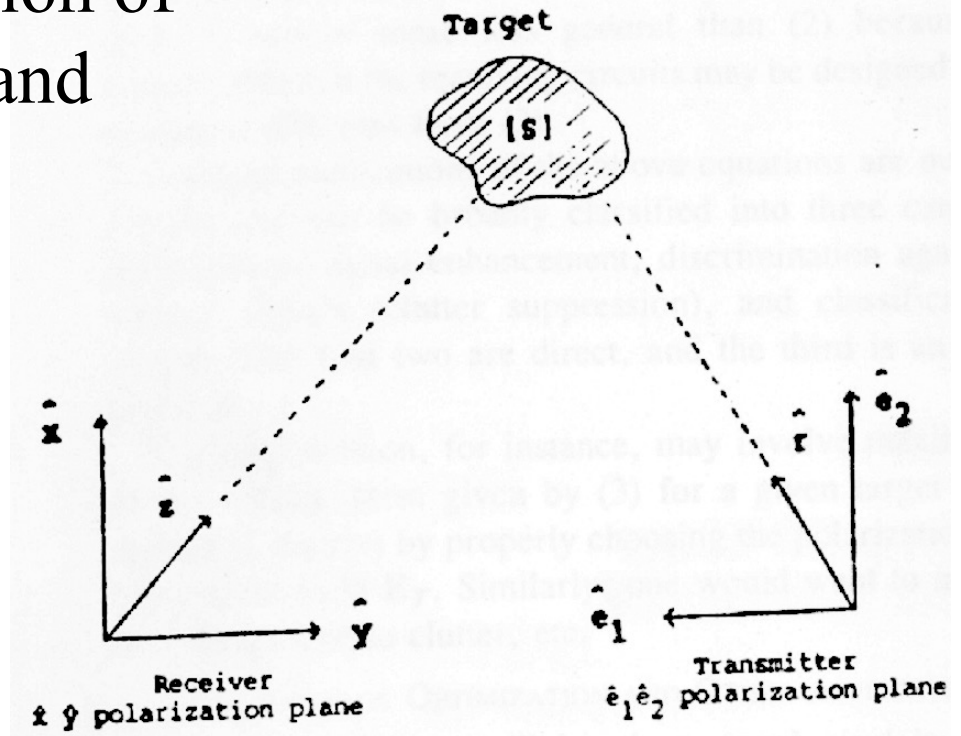
- The polarisation is a combination of the horizontal and vertical fields and could be represented by an ellipse (locus of the end point of the electric field)



# Imaging system

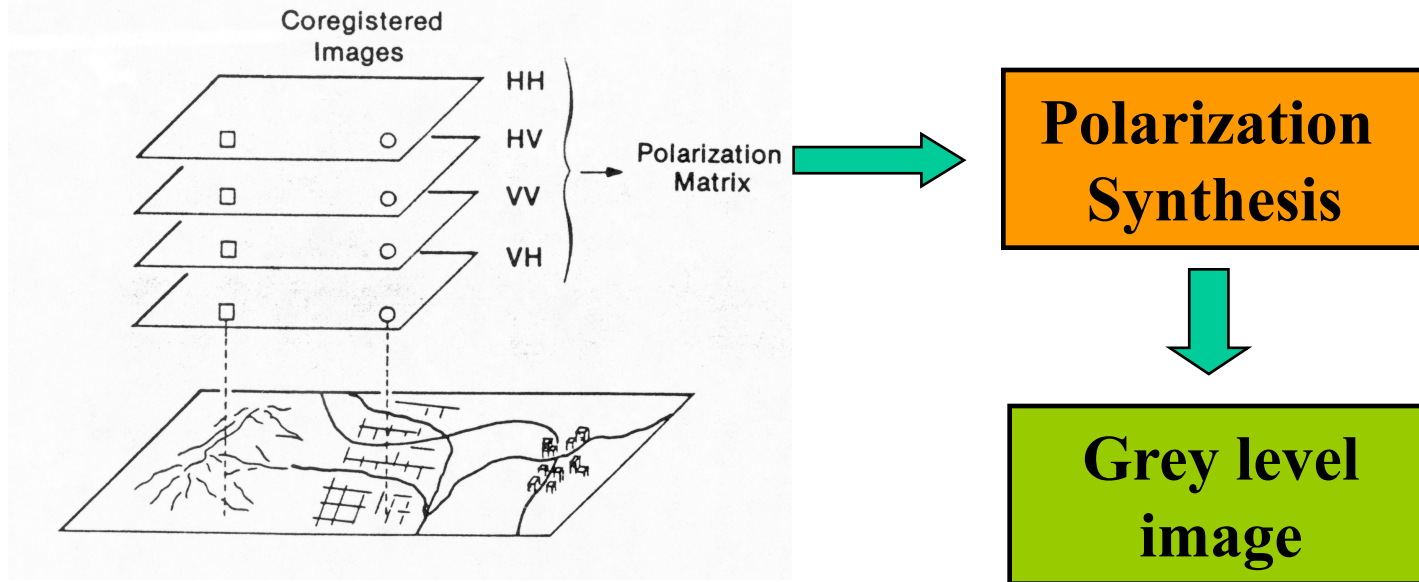
Consider the polarization of the transmitter and the receiver

		receiver	
		h	v
transmitter	h	hh	hv
	v	vh	vv





# Polarization Synthesis



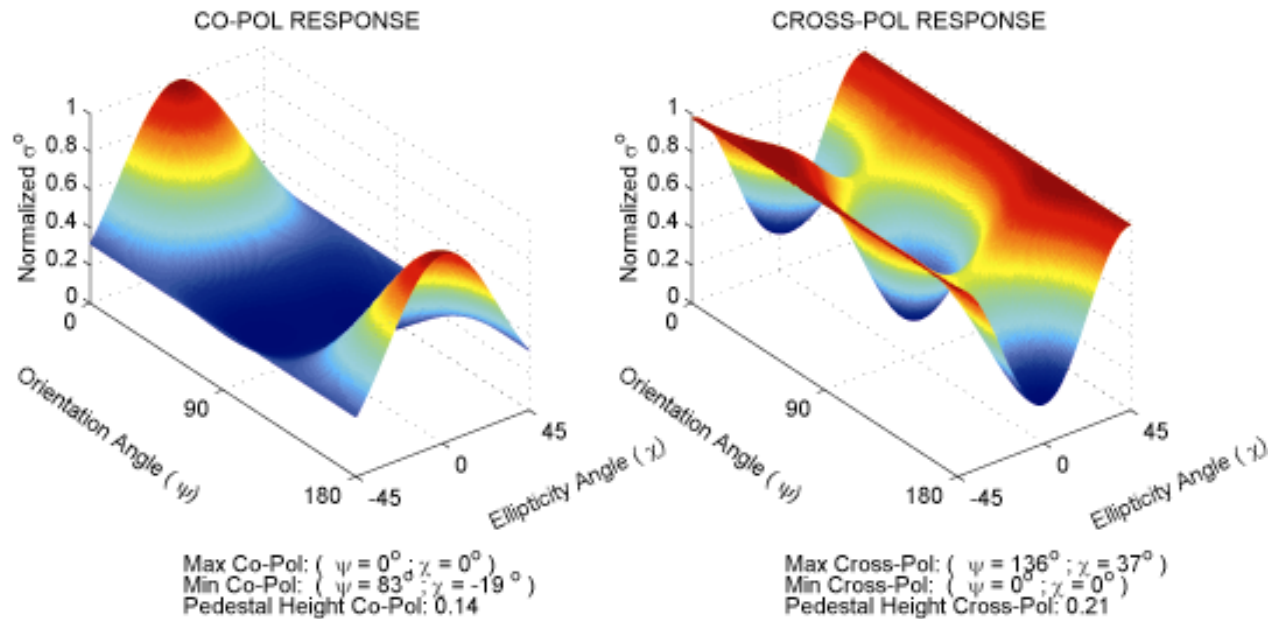
Chouse transmit and receive polarizations  
→ produce one grey level image



# Polarization signatures of a forest area in Shirley Bay

Line 4 Pass 1 9-NOV-1999 Calibrated

Linear Pol (dB):  $\sigma_{HH}^0 = 23.50$  ;  $\sigma_{HV}^0 = 12.04$  ;  $\sigma_{VV}^0 = 15.64$   
Circular Pol (dB):  $\sigma_{RR}^0 = 18.48$  ;  $\sigma_{LR}^0 = 18.79$  ;  $\sigma_{LL}^0 = 18.49$



Incident angle:  $39.10^\circ$

Target coord: L-(500 - 600) ; P-(7100 - 7300)



# POLARIMETRIC SAR IMAGE

**Multi-channel image – 3 complex elements**

$$x = \begin{bmatrix} hh \\ hv \\ vv \end{bmatrix}$$

each element has  
a zero mean circular  
gaussian distribution

**Complex gaussian pdf** ( $\Sigma$  is the covariance matrix)

$$p(x | \Sigma) = \frac{1}{\pi^3 |\Sigma|} \exp(-x^* \Sigma^{-1} x)$$

$x^*$  is the complex conjugate transpose of  $x$

## Multi-look images

Use the covariance matrix

$$\hat{\Sigma} = C = \frac{1}{n_s} \sum_{x \in S} x x^*$$

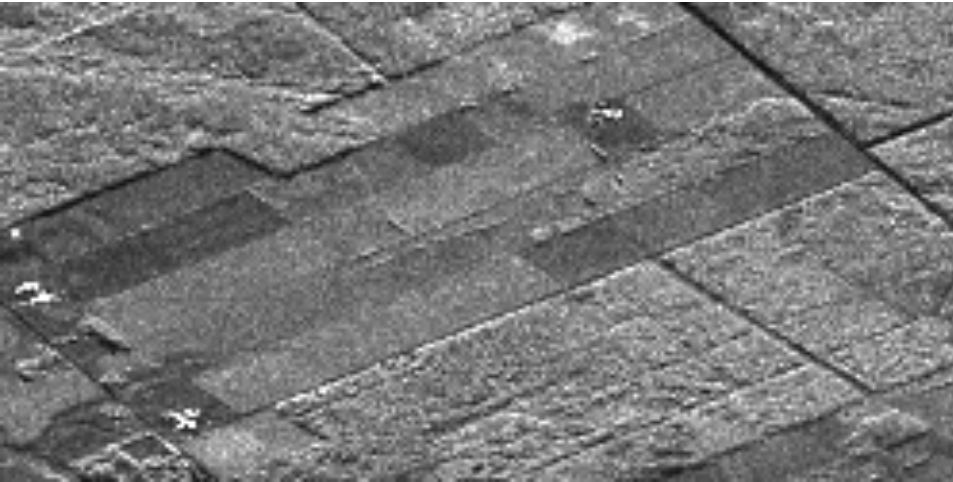
$n_s$  is the number of pixels  
in segment  $s$

$$C = \frac{1}{n} \begin{bmatrix} \sum hh & \sum hh & \sum hh \\ \sum hv & \sum hv & \sum hv \\ \sum vv & \sum vv & \sum vv \end{bmatrix}$$

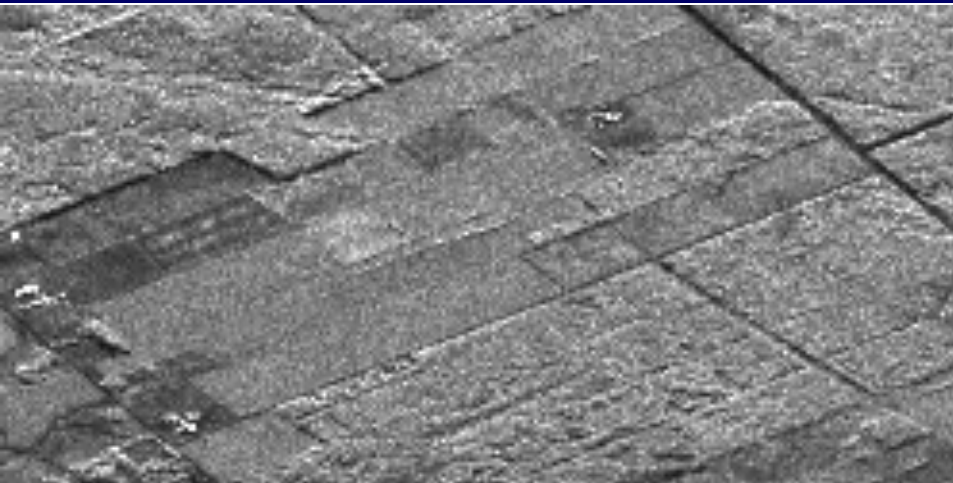
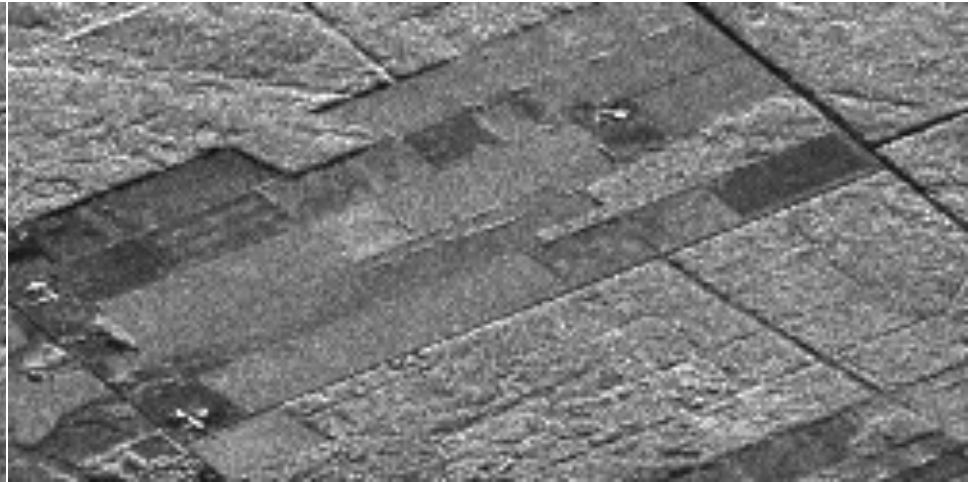
Follows a complex Wishart distribution

# Amplitude values

**|hh|**



**|hv|**



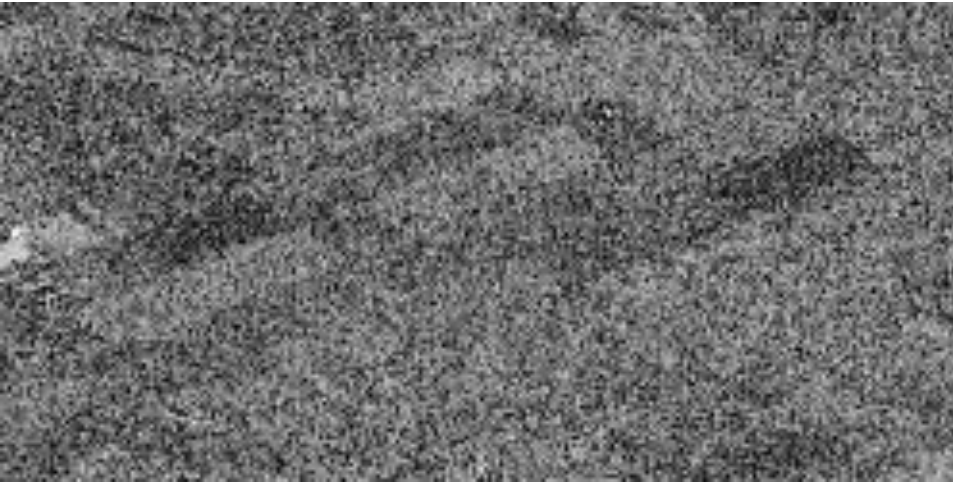
**|vv|**

**|hh| / |hv| / |vv|**

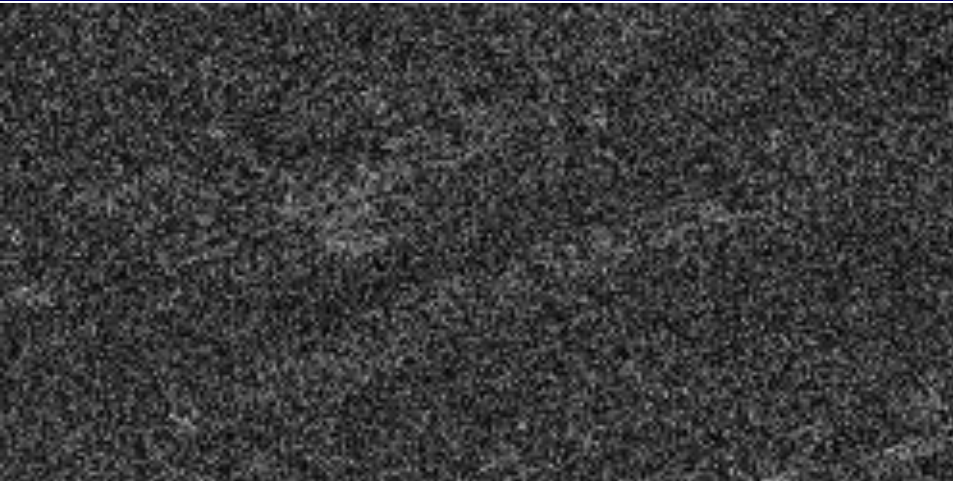
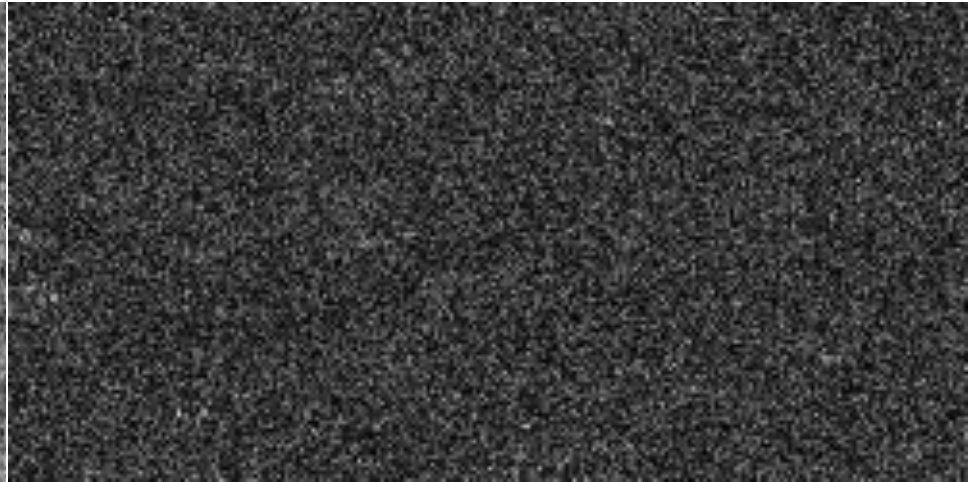
**80 pixels / cell**

# Correlation – module (0 – 1)

**hh vv\***



**hh hv\***



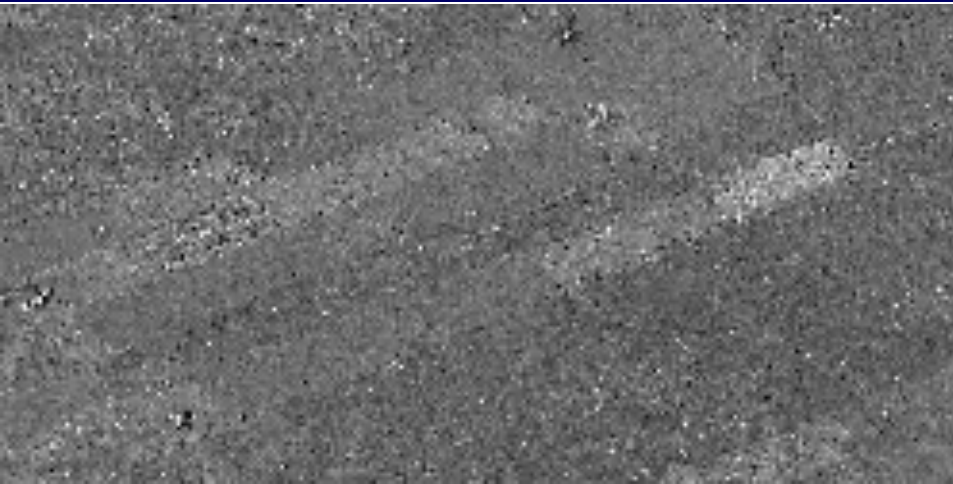
**vv hv\***



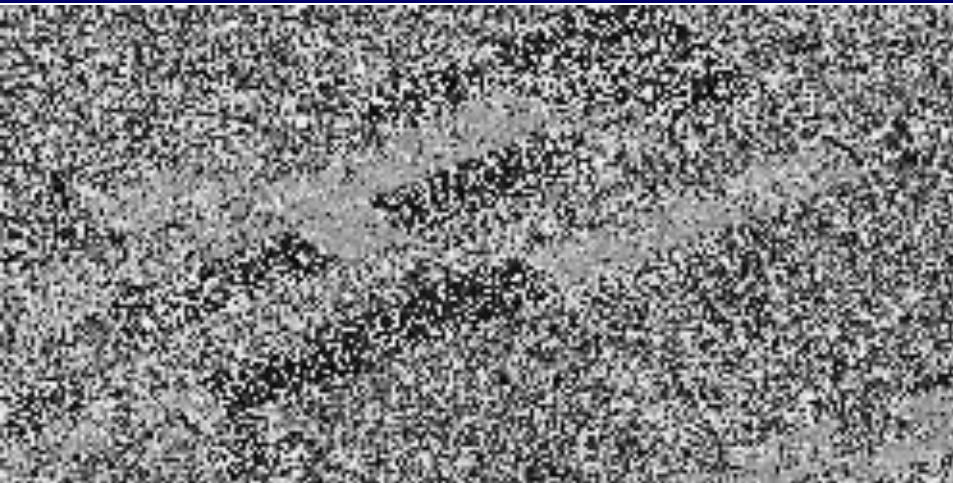
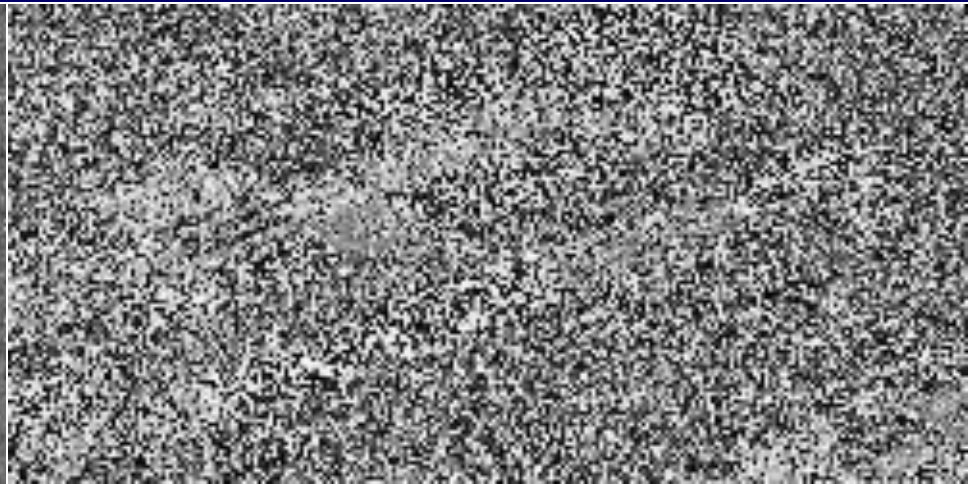
**hh vv\*/ hh hv\*/ vv hv\***

# Correlation – phase (-180° – 180°)

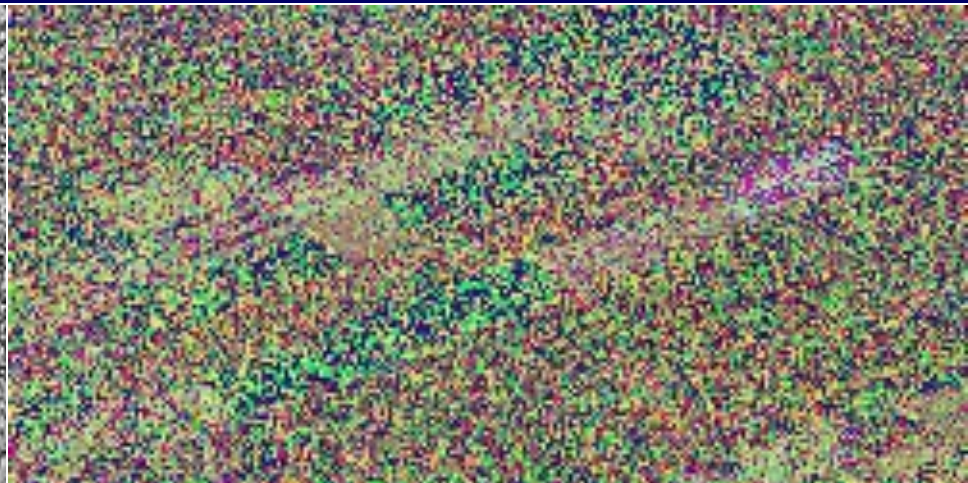
**hh vv\***



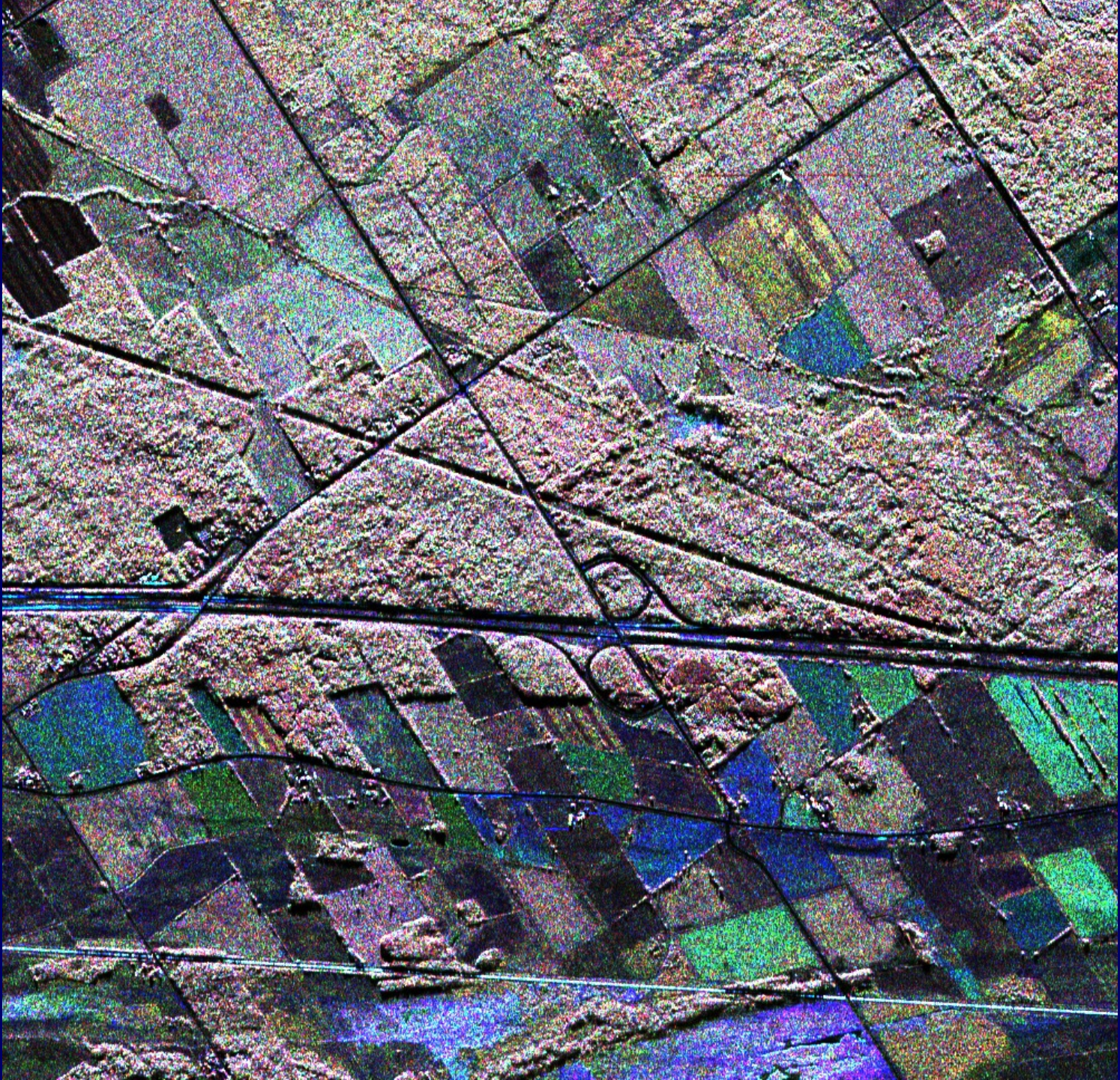
**hh hv\***



**vv hv\***



**hh vv\* / hh hv\* / vv hv\***



Amplitude image

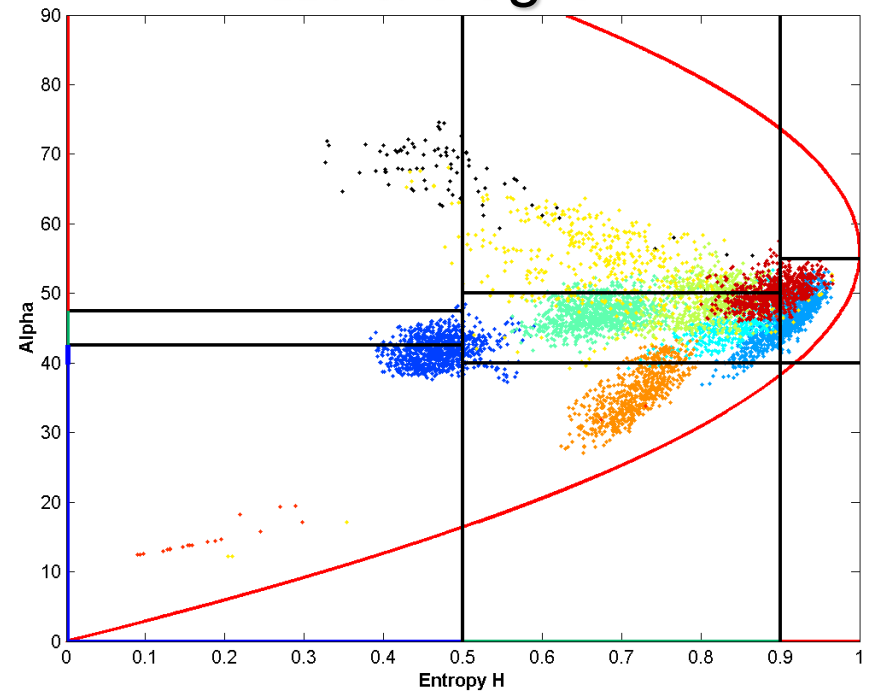


# Cloude Decomposition

SPAN image



H /  $\alpha$  diagram



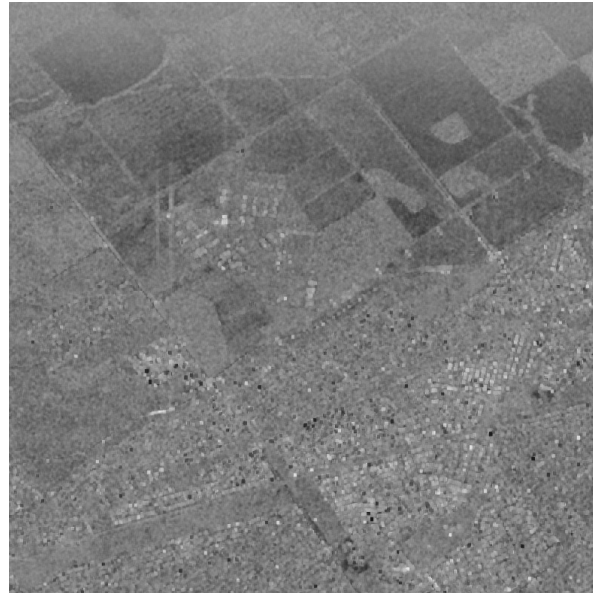
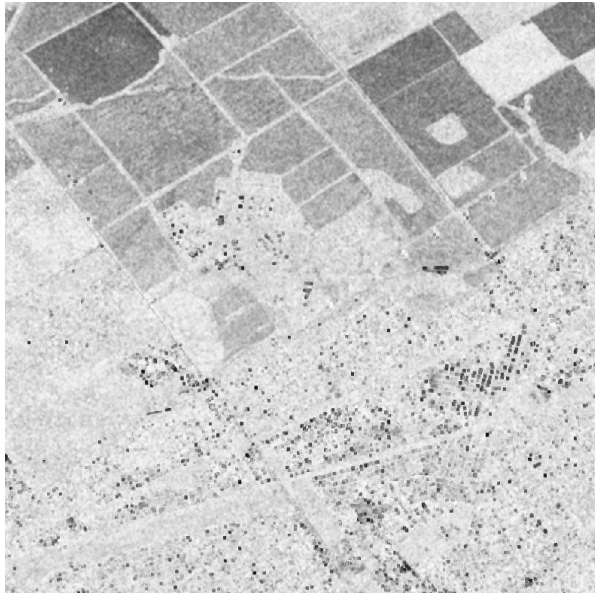


## Clouds decomposition ...

Entropy

$\alpha$

Anisotropy

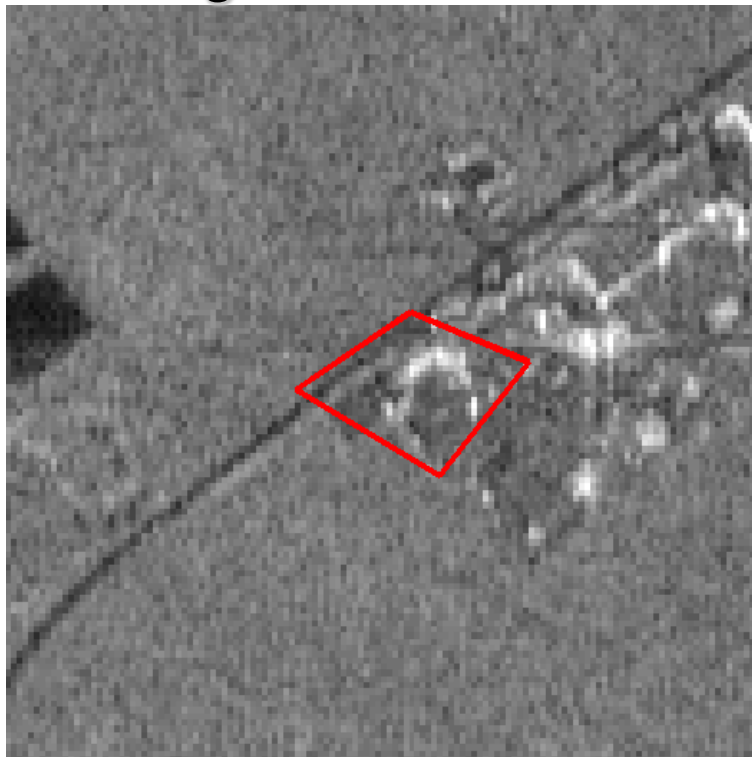




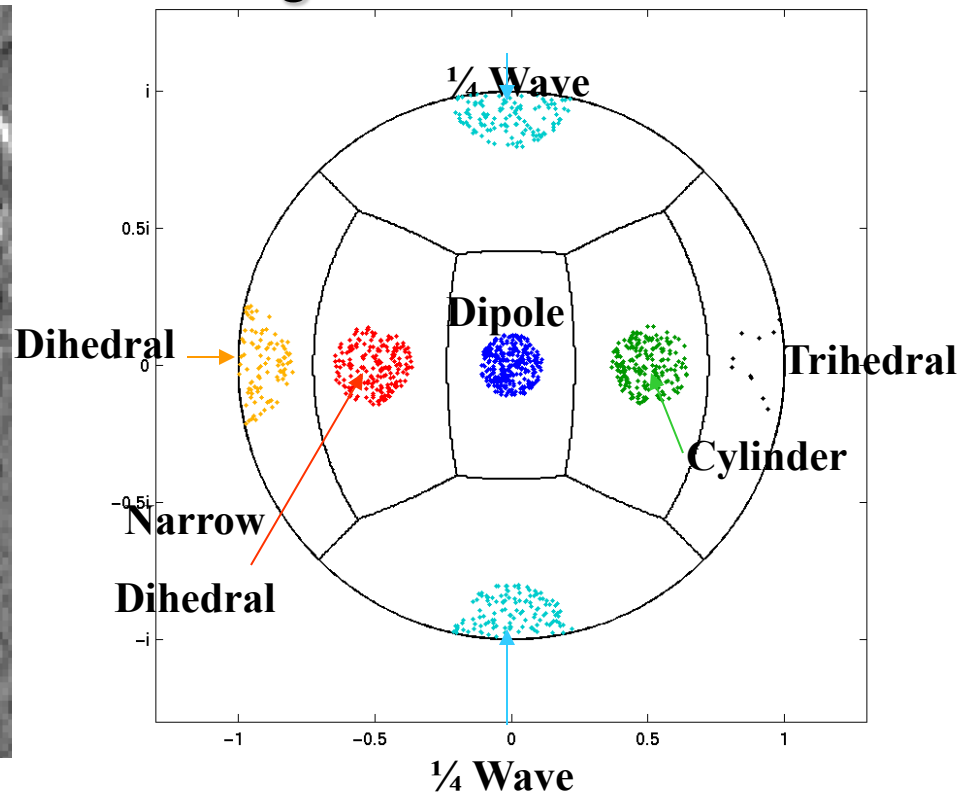


# Cameron Decomposition

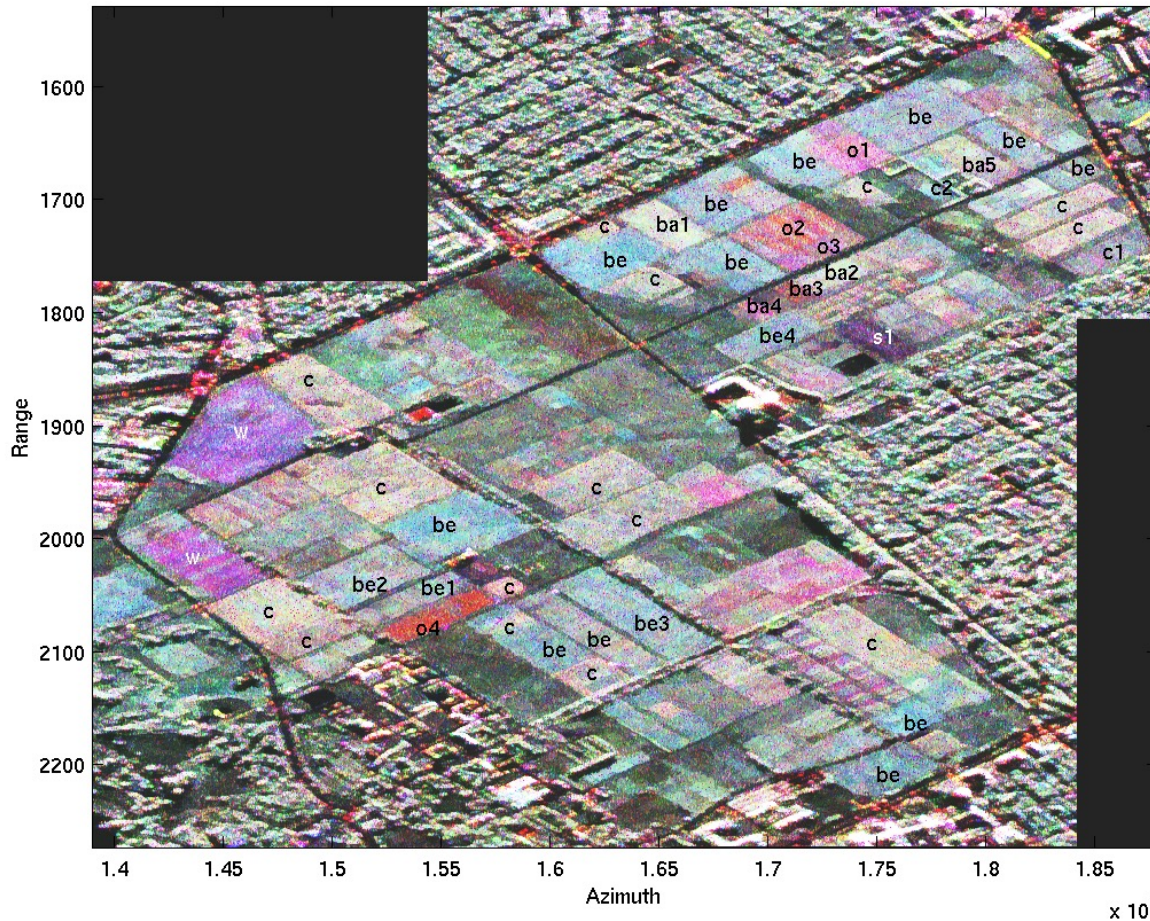
## Region of Interest



## Z Diagram Threshold 2dB



# Classification of Distributed Target



**w : wheat**

**c : corn**

**be: beans**

**o : oats**

**ba: barley**

**s : bare soil**

.....  
**o1: rows of 90cm oats (4m) & soil (1.5m)**

**o2: rows of cut (10m) & 90cm (28m) oats**

**o3: rows of 90cm oats (1m) & grass (3m)**

**o4: oats, uniform, ripe (90cm)**

**c1: corn, young leaves only**

**c2: corn, unhealthy, sparse, grass**

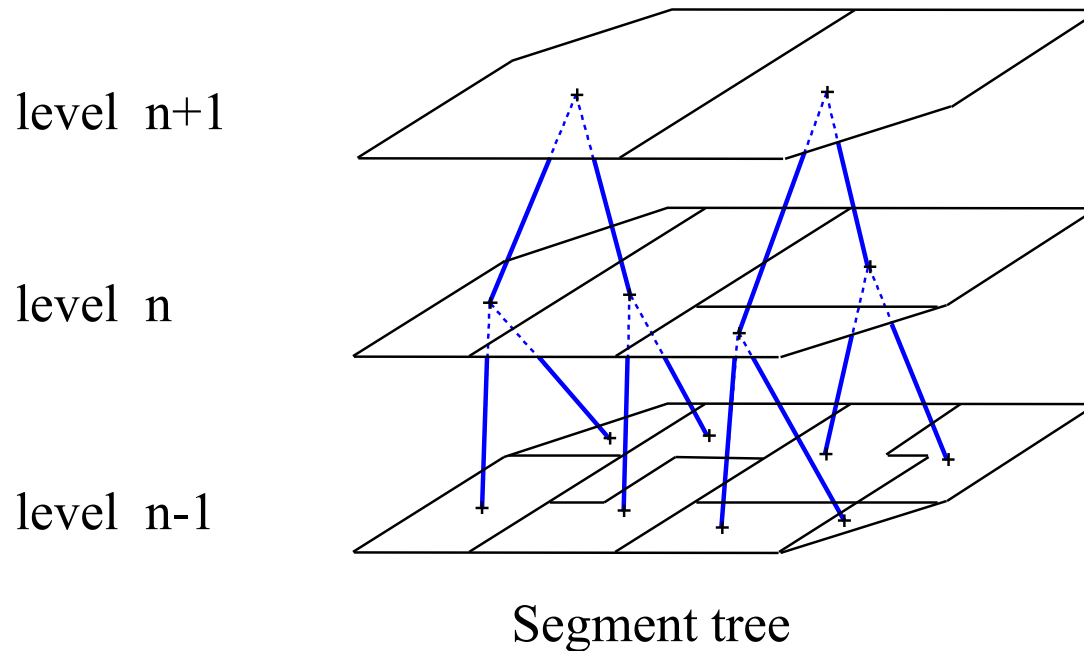
**ba1: barley, ripe**

**ba2: barley, young & green**

.....  
**polarimetric filter : Lee phase preserving**

# HIERARCHICAL SEGMENTATION BY STEP-WISE OPTIMISATION

A hierarchical segmentation begins with an initial partition  $P^0$  (with  $N$  segments) and then sequentially merges these segments.



# SEGMENTATION AS MAXIMUM LIKELIHOOD APPROXIMATION

1) need a partition of the image

$$P = \{s_k\}, \quad s_k = \{i\} \subset I$$

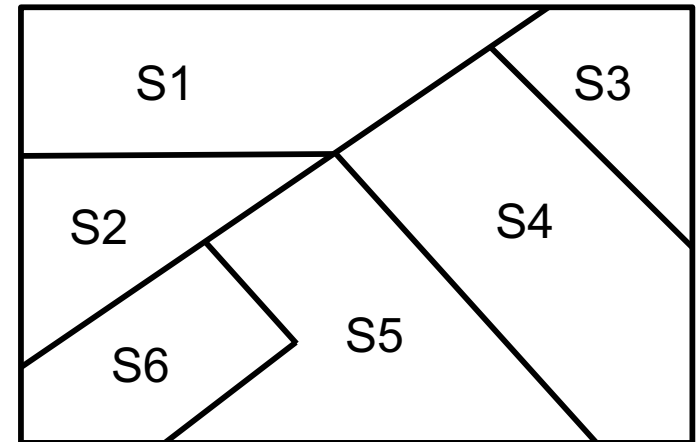
2) need statistical parameters

$$\theta = \{\theta_s\}, \quad s \in P$$

3) need an image probability model

$$p(x_i | \theta_s)$$

$x_i$  are conditionally independent

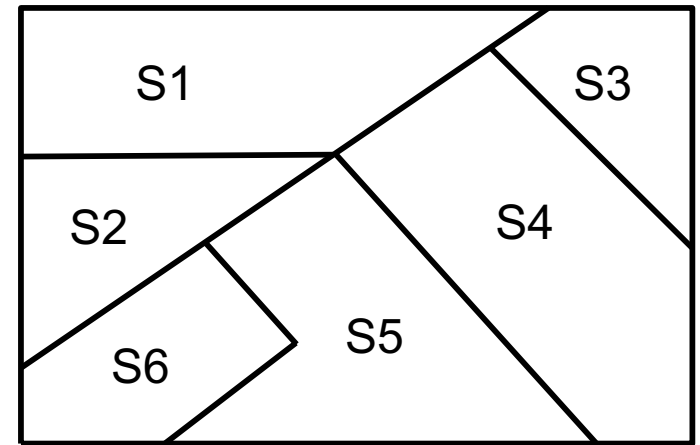


Given an image  $X = \{x_i\}$ ,  $i \in I$

the likelihood of  $\theta = \{\theta_s\}$ ,  $P$

is  $L(\theta, P | X) = p(X | \theta, P)$

$$L(\theta, P | X) = \prod_{i \in I} p(x_i | \theta_{s(i)}) \Big|_P$$

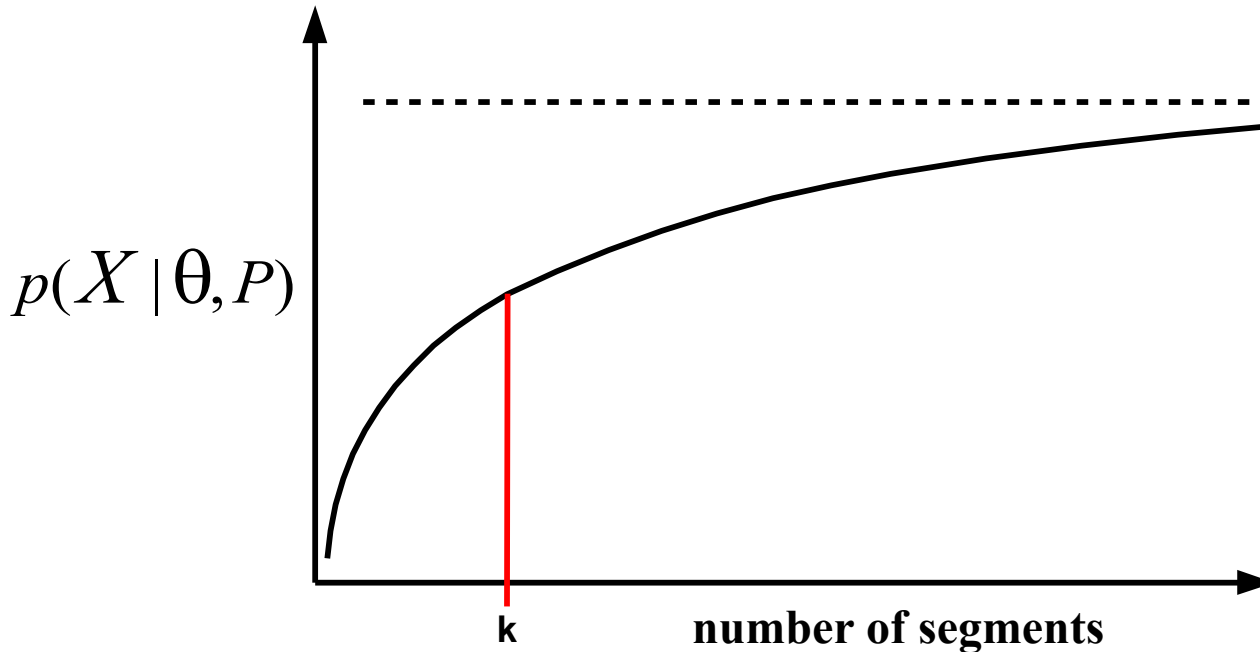


**The segmentation problem is to find the partition that maximizes the likelihood.**

**Global search – too many possible partitions.**

$\theta_s$  is derived from statistics calculated over a segment  $s$ .

**The maximum likelihood increases with the number of segments**



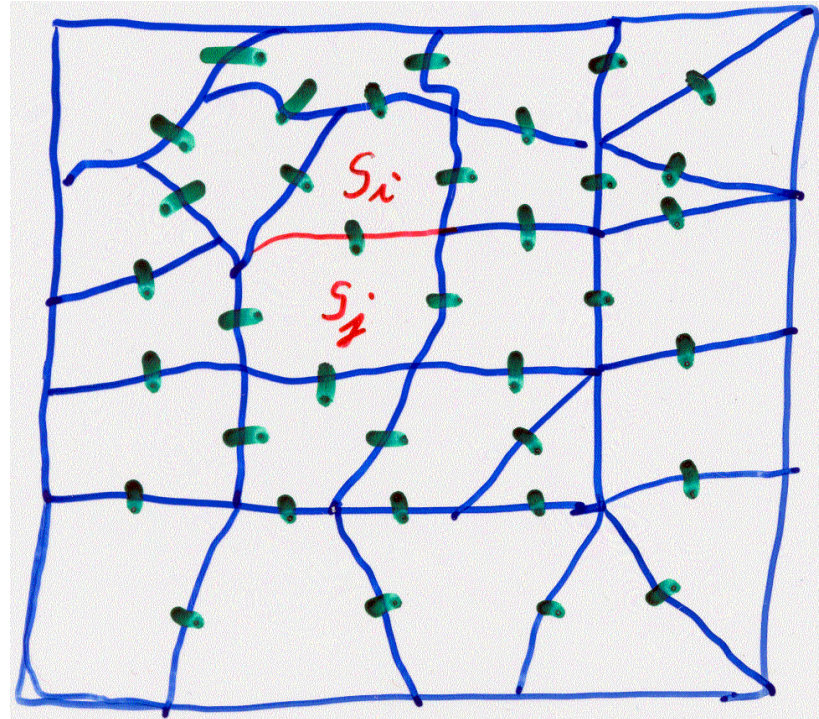
**Can't find the optimum partition with  $k$  segments,  $P_k$   
Too many, except for  $P_1$  and  $P_{n \times n}$ .**

**Hierarchical segmentation**

**$\rightarrow$  get  $P_k$  from  $P_{k+1}$  by merging 2 segments.**

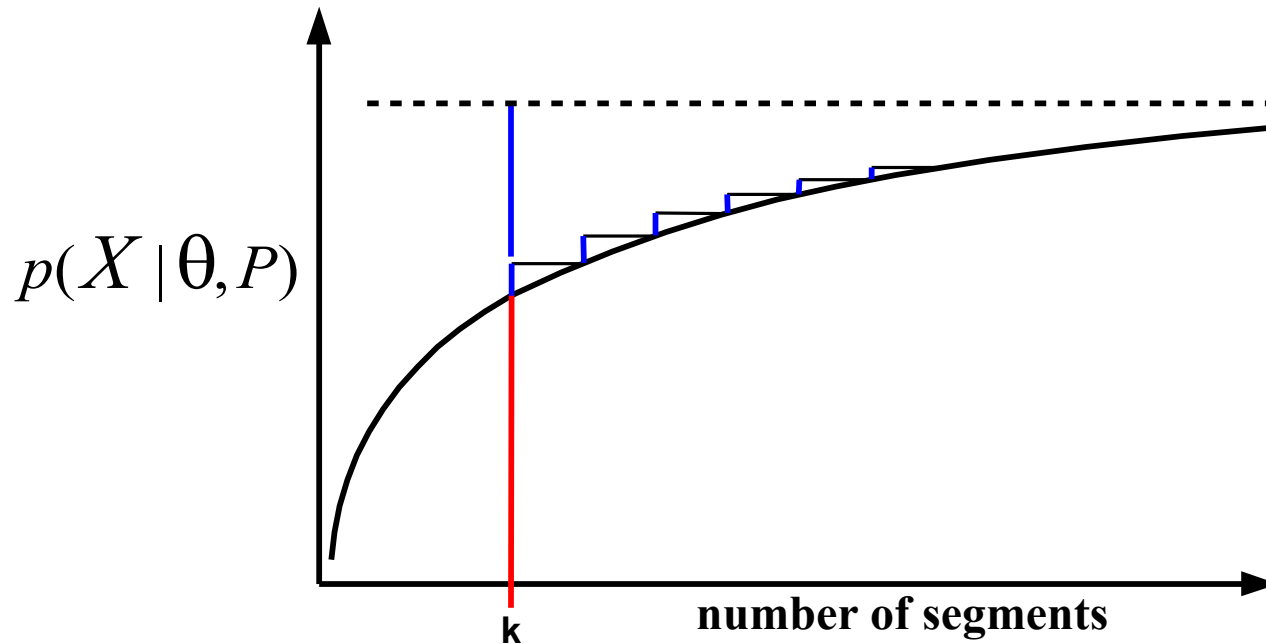
## Stepwise optimization

- examine each adjacent segment pair
- merge the pair that minimizes the criterion



## Merging criterion:

merge the 2 segments producing the smallest decrease of the maximum likelihood  
(stepwise optimization)



**Sub-optimum within hierarchical merging framework.**



## Log likelihood form

$$\ln(L(\theta, P | X)) = \ln\left(\prod_{i \in I} p(x_i | \theta_{s(i)})\right) = \sum_{i \in I} \ln(p(x_i | \theta_{s(i)}))$$

## Summation inside region

$$\sum_{s \in P} \sum_{i \in s} \ln(p(x_i | \theta_s)) = \sum_{s \in P} LML(s)$$

## Criterion $\rightarrow$ cost of merging 2 segments

$$\Delta = LML(s_i) + LML(s_j) - LML(s_i \cup s_j)$$

$$\Delta = \sum_{x \in s_i} \ln(p(x | \theta_{s_i})) + \sum_{x \in s_j} \ln(p(x | \theta_{s_j})) - \sum_{x \in s_i \cup s_j} \ln(p(x | \theta_{s_i \cup s_j}))$$

**minimize**  $|\Delta|$

# POLARIMETRIC SAR IMAGE

**Multi-channel image – 3 complex elements**

$$x = \begin{bmatrix} hh \\ hv \\ vv \end{bmatrix}$$

each element has  
a zero mean circular  
gaussian distribution

**Complex gaussian pdf** ( $\Sigma$  is the covariance matrix)

$$p(x | \Sigma) = \frac{1}{\pi^3 |\Sigma|} \exp(-x^* \Sigma^{-1} x)$$

$x^*$  is the complex conjugate transpose of  $x$

**The best maximum likelihood estimate of  $\Sigma$  is  
the covariance calculated over the region (segment)**

$$\hat{\Sigma} = C = \frac{1}{n_s} \sum_{x \in S} x x^*$$

$n_s$  is the number of pixels  
in segment  $s$

$$C = \frac{1}{n} \begin{bmatrix} \sum hh & hh^* & \sum hh & hv^* & \sum hh & vv^* \\ \sum hv & hh^* & \sum hv & hv^* & \sum hv & vv^* \\ \sum vv & hh^* & \sum vv & hv^* & \sum vv & vv^* \end{bmatrix}$$

***LML* for a region  $s$  is**

$$\begin{aligned} LML(s) &= \sum_{x \in S} \ln(p(x | C_s)) = \sum_{x \in S} \ln \left( \frac{1}{\pi^3 |C_s|} \exp(-x^* C_s^{-1} x) \right) \\ &= \sum_{x \in S} \left[ -\ln \pi^3 - \ln |C_s| - x^* C_s^{-1} x \right] \\ &= -n_s \ln \pi^3 - n_s \ln |C_s| - \sum_{x \in S} x^* C_s^{-1} x \\ &= -n_s \ln |C_s| - n_s \ln \pi^3 - 3n_s \end{aligned}$$

constant term for the whole image

**The variation produced by merging 2 segments is**

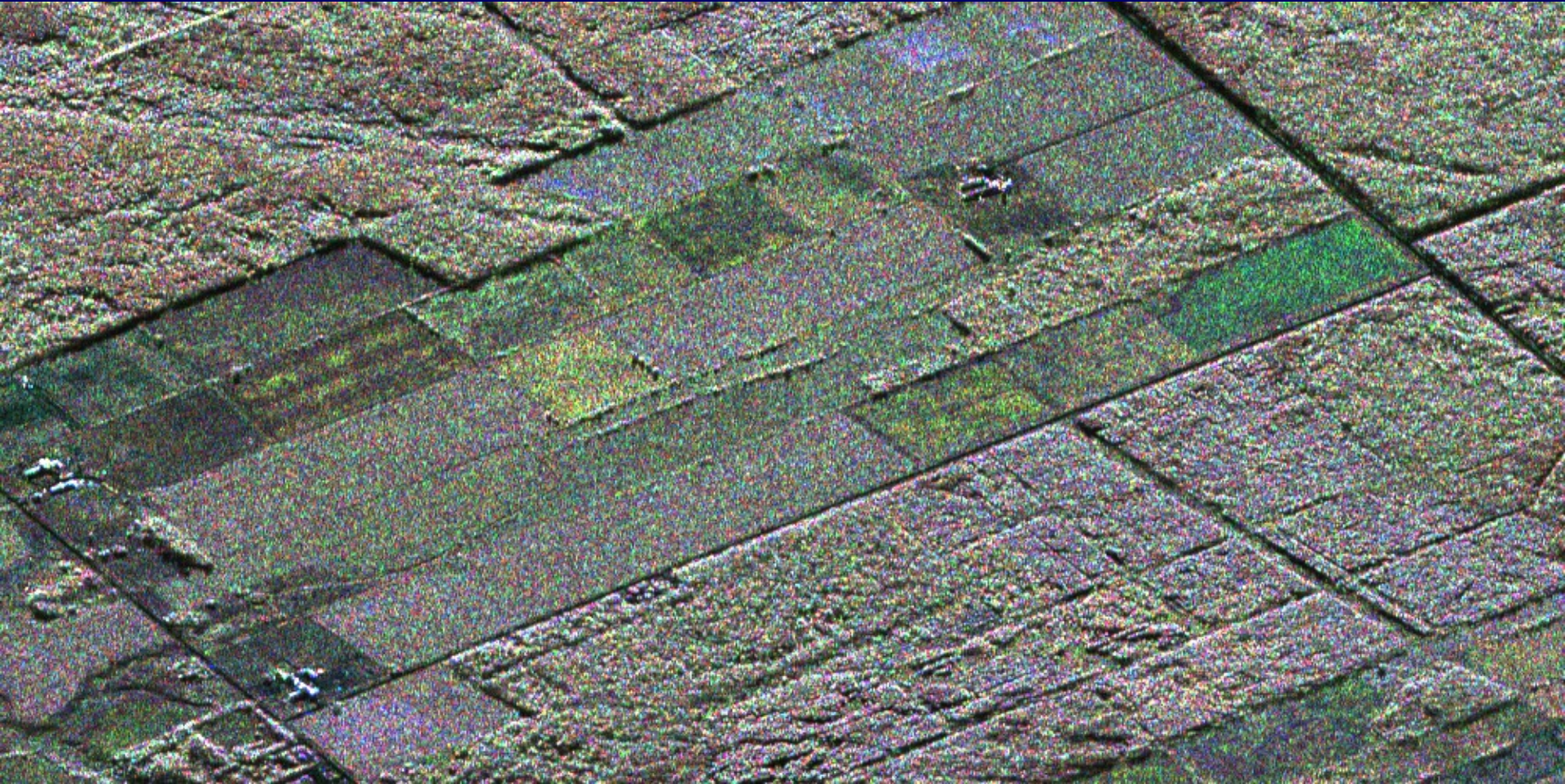
$$\begin{aligned}\Delta &= LML(s_i) + LML(s_j) - LML(s_i \cup s_j) \\ &= -n_{si} \ln |C_{si}| - n_{sj} \ln |C_{sj}| + (n_{si} + n_{sj}) \ln |C_{si \cup sj}|\end{aligned}$$

**Hierarchical segmentation:**

**at each iteration, merge the 2 segments  
that minimize the stepwise criterion  $C_{i,j}$**

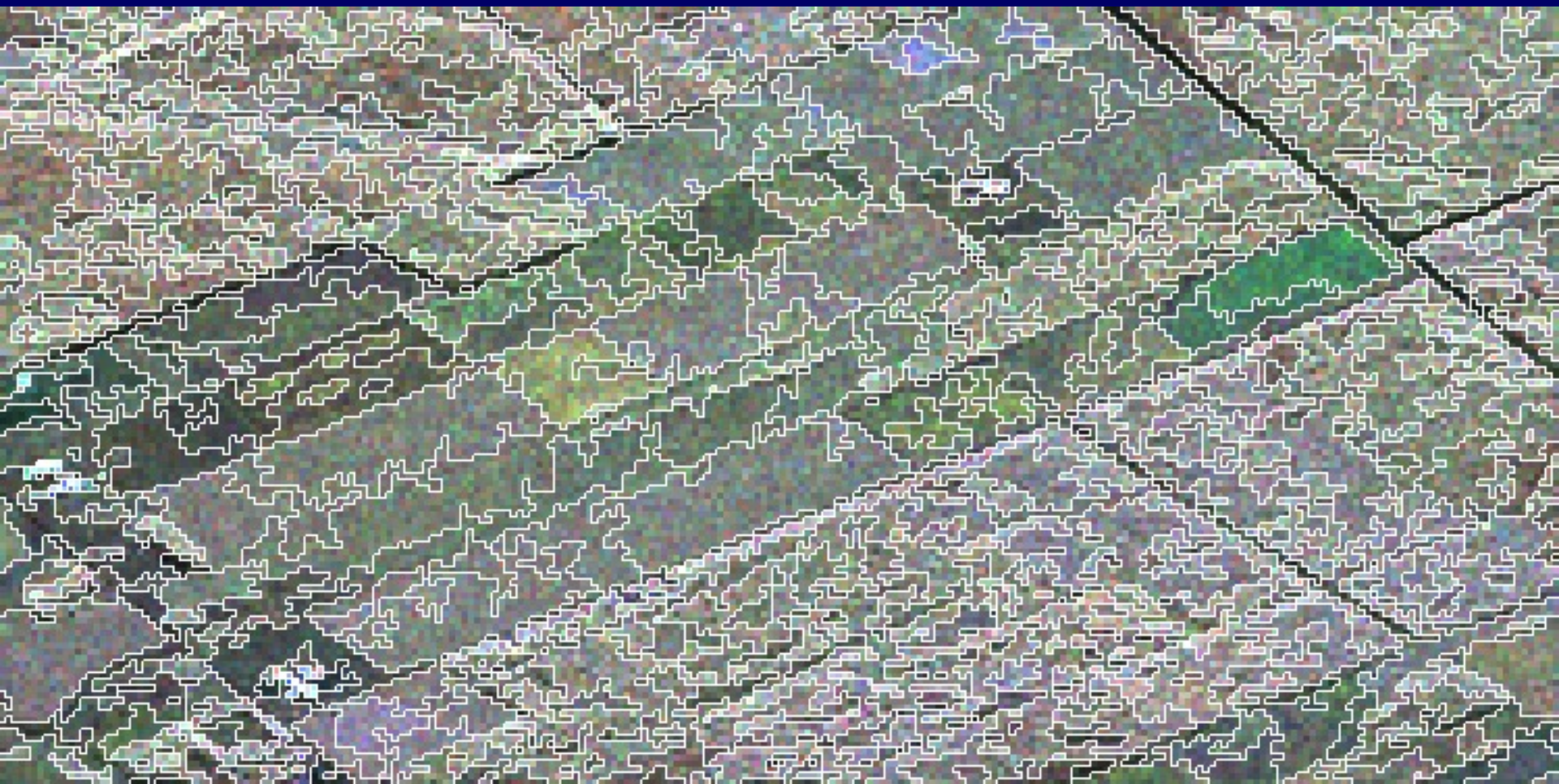
$$C_{i,j} = (n_{si} + n_{sj}) \ln |C_{si \cup sj}| - n_{si} \ln |C_{si}| - n_{sj} \ln |C_{sj}|$$

# Amplitude image

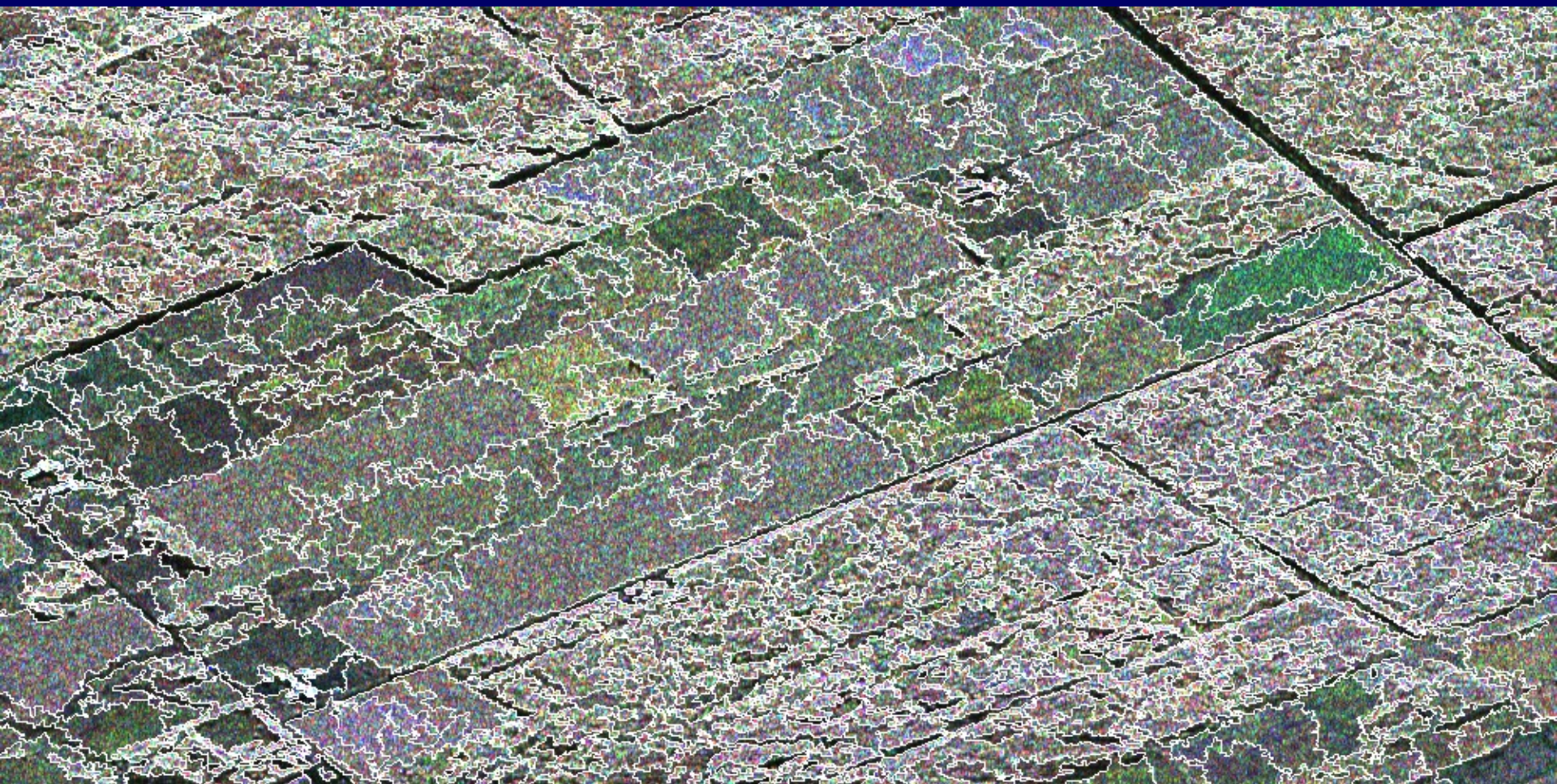


5 pixels / cell

**1000 segments – low resolution**

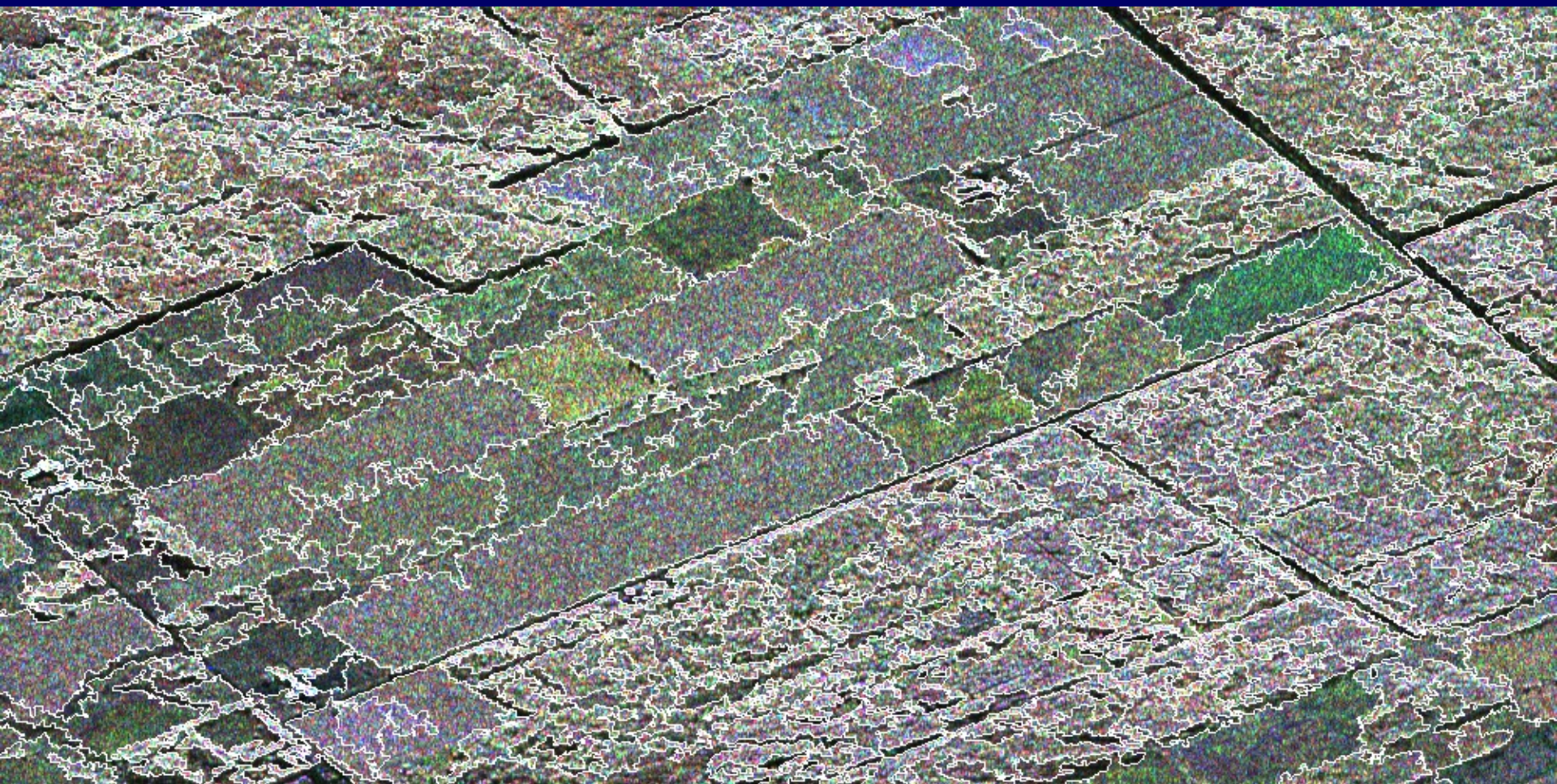


**1000 segments**

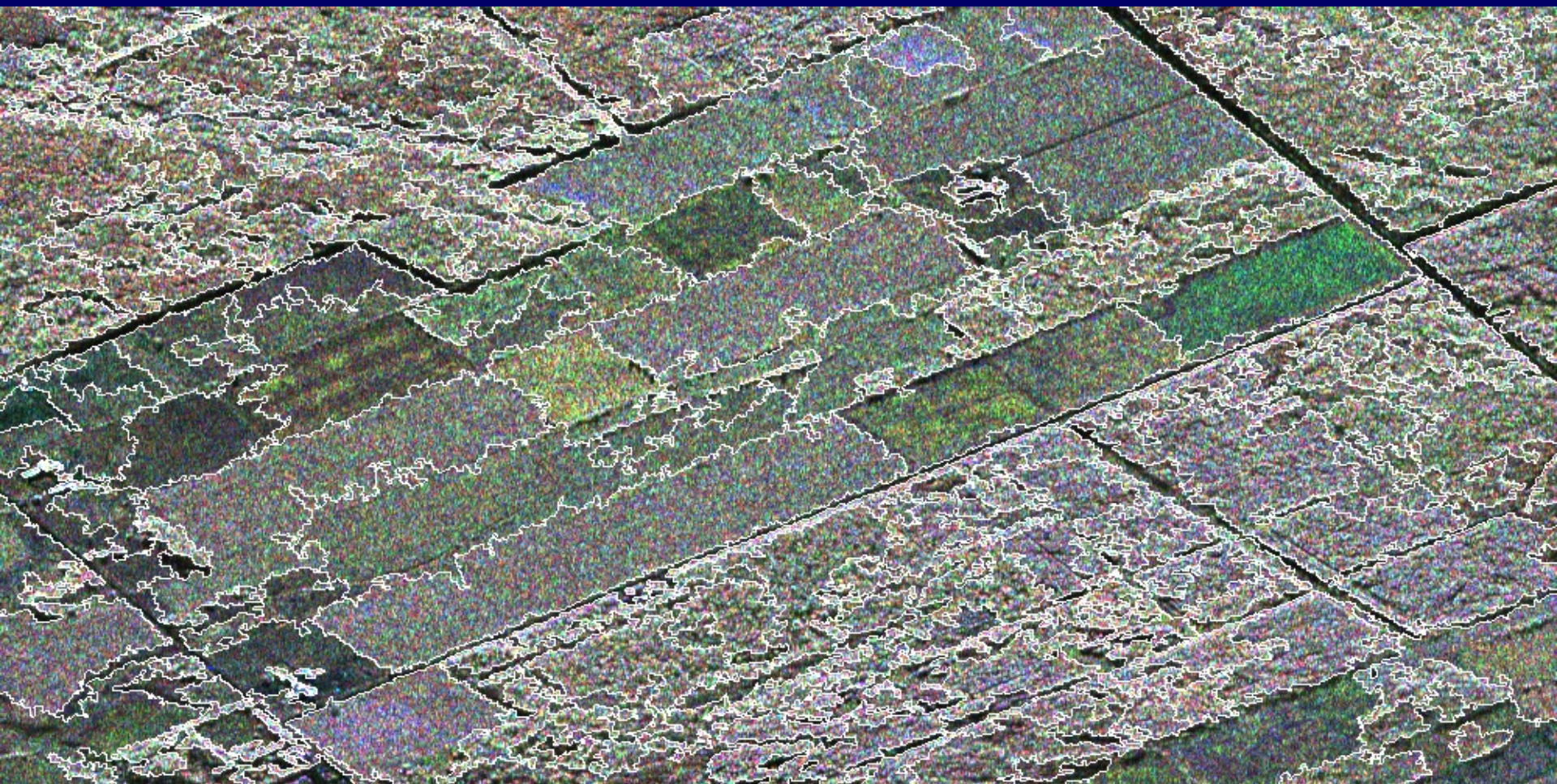




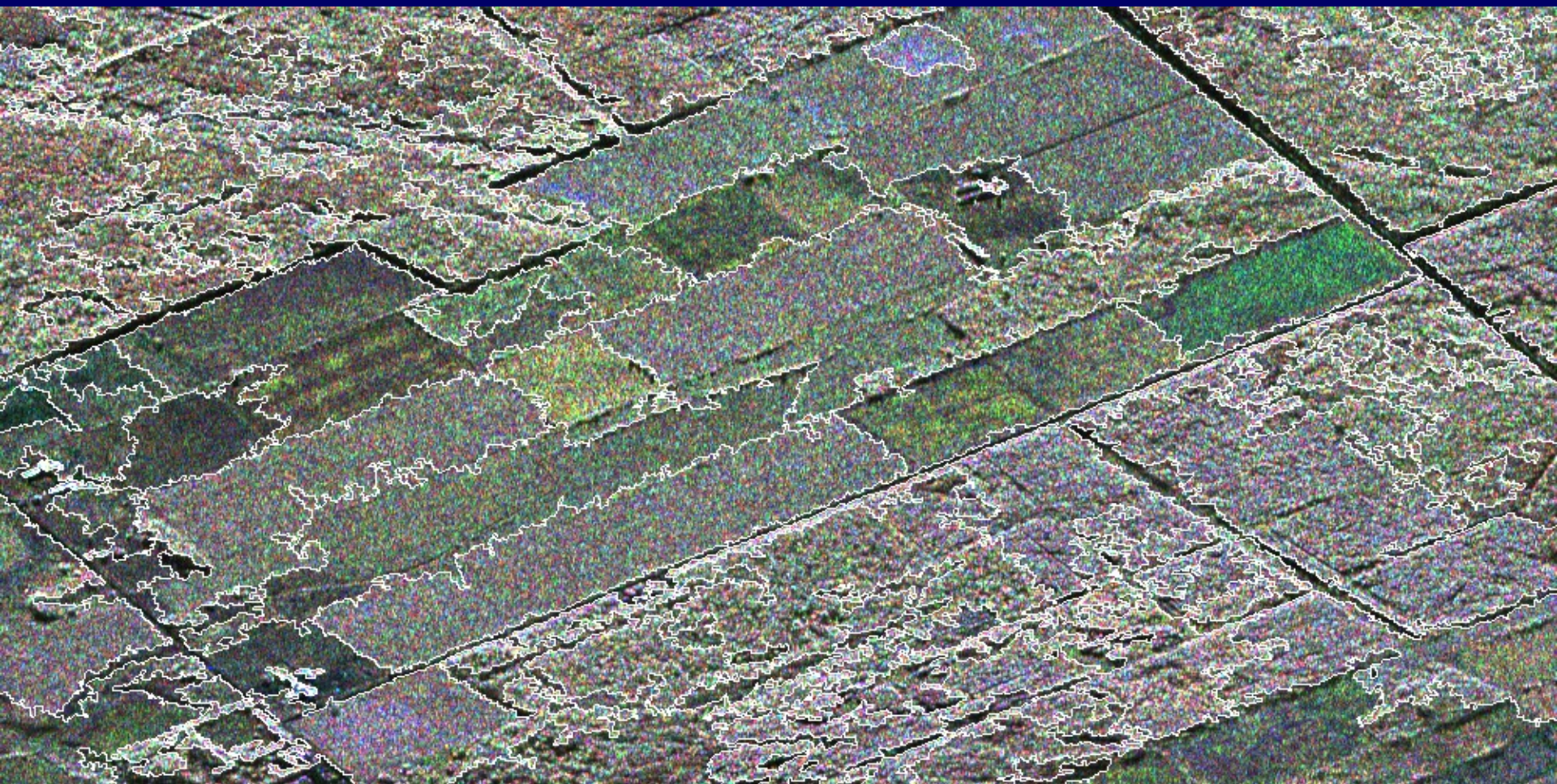
**500 segments**



**200 segments**



**100 segments**



# CONCLUSION

- Hierarchical segmentation produces good results
- Criterion should be adapted to the application
- Good polarimetric criterion
- The first merges should be done correctly

# CRITERION FOR SMALL SEGMENTS

The determinant  $|\mathbf{C}|$  is null for small segments

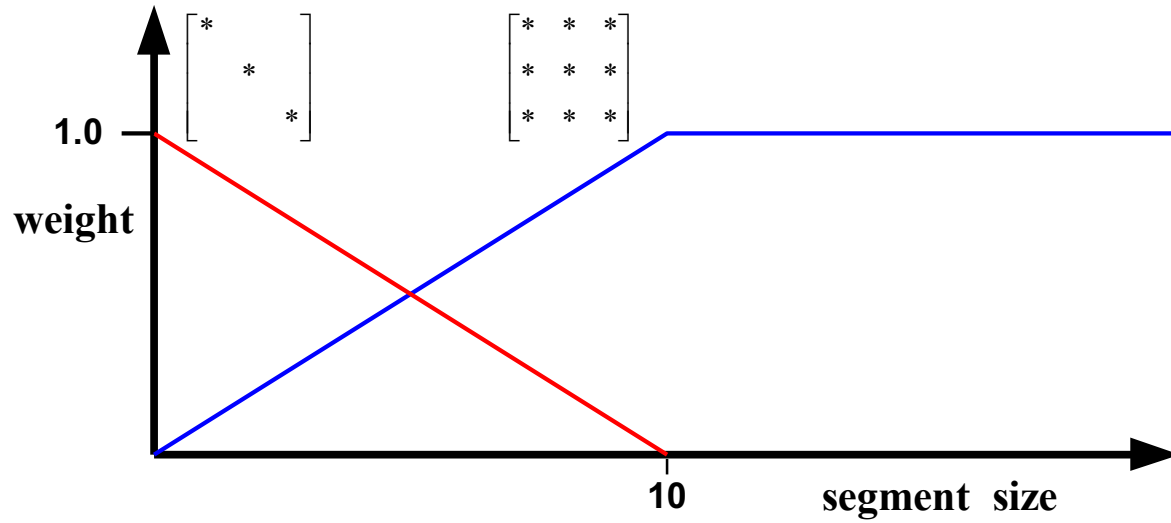
$$\mathbf{C} = \frac{1}{n} \begin{bmatrix} \sum hh \, hh^* & \sum hh \, hv^* & \sum hh \, vv^* \\ \sum hv \, hh^* & \sum hv \, hv^* & \sum hv \, vv^* \\ \sum vv \, hh^* & \sum vv \, hv^* & \sum vv \, vv^* \end{bmatrix}$$

Reduce covariance matrix model for small segments

$$\frac{1}{n} \begin{bmatrix} \sum hh \, hh^* & 0 & \sum hh \, vv^* \\ 0 & \sum hv \, hv^* & 0 \\ \sum vv \, hh^* & 0 & \sum vv \, vv^* \end{bmatrix}$$

$$\frac{1}{n} \begin{bmatrix} \sum hh \, hh^* & 0 & 0 \\ 0 & \sum hv \, hv^* & 0 \\ 0 & 0 & \sum vv \, vv^* \end{bmatrix}$$

# Gradual transition between models



# SEGMENT SHAPE CRITERIA

**High speckle noise**

**→ first merges produce ill formed segments**

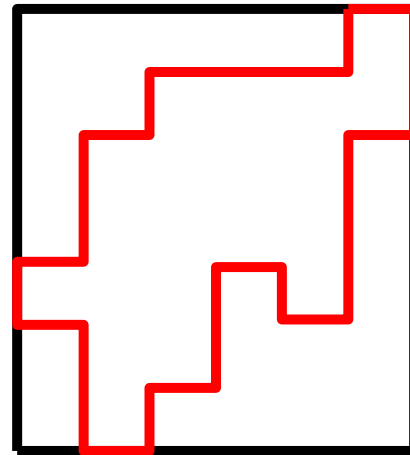
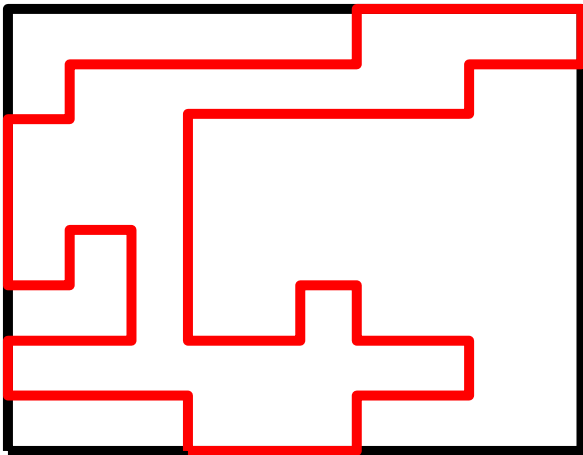
- Bonding box – perimeter       $C_p$
- Bonding box – area       $C_a$
- Contour length       $C_l$

**New criteria**

$$C_{i,j}^{contour} = C_{i,j}^{polar} \times C_p^2 \times C_a \times C_l$$

## Bonding box – perimeter

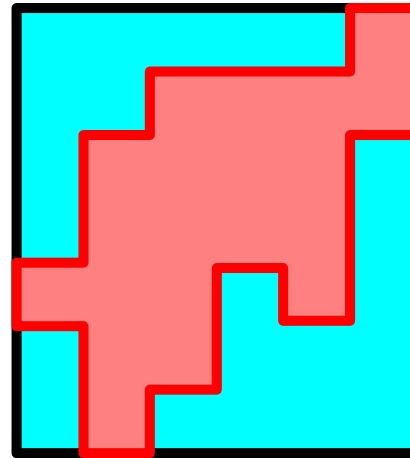
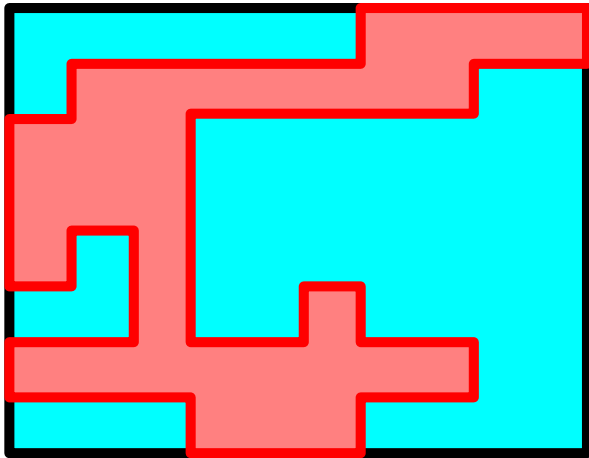
$$C_p = \frac{\text{perimeter of } S_i \cup S_j}{\text{perimeter of bonding box}}$$





## Bonding box – area

$$Ca = \frac{\text{area of bonding box}}{\text{area of } S_i \cup S_j}$$

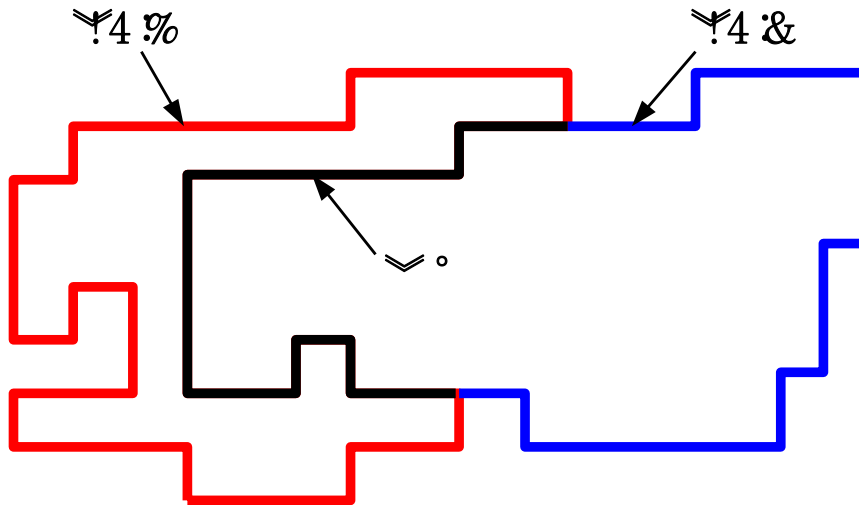


## Contour length

$L_c$  = length of common part of contours

$Lex_i$  = length of exclusive part for  $S_i$

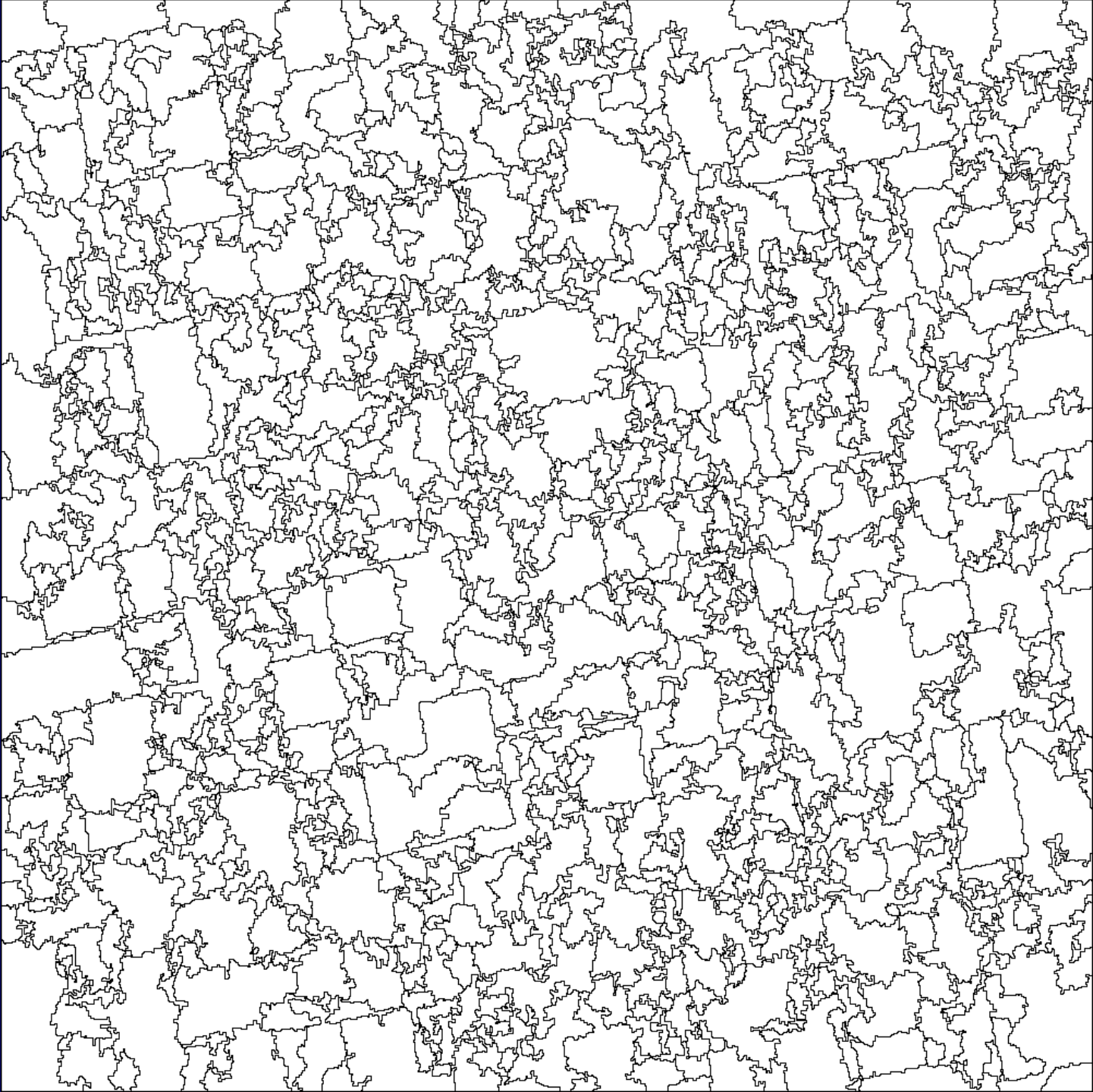
$$Cl = \text{Min} \left\{ \frac{Lex_i}{L_c}, \frac{Lex_j}{L_c} \right\}$$



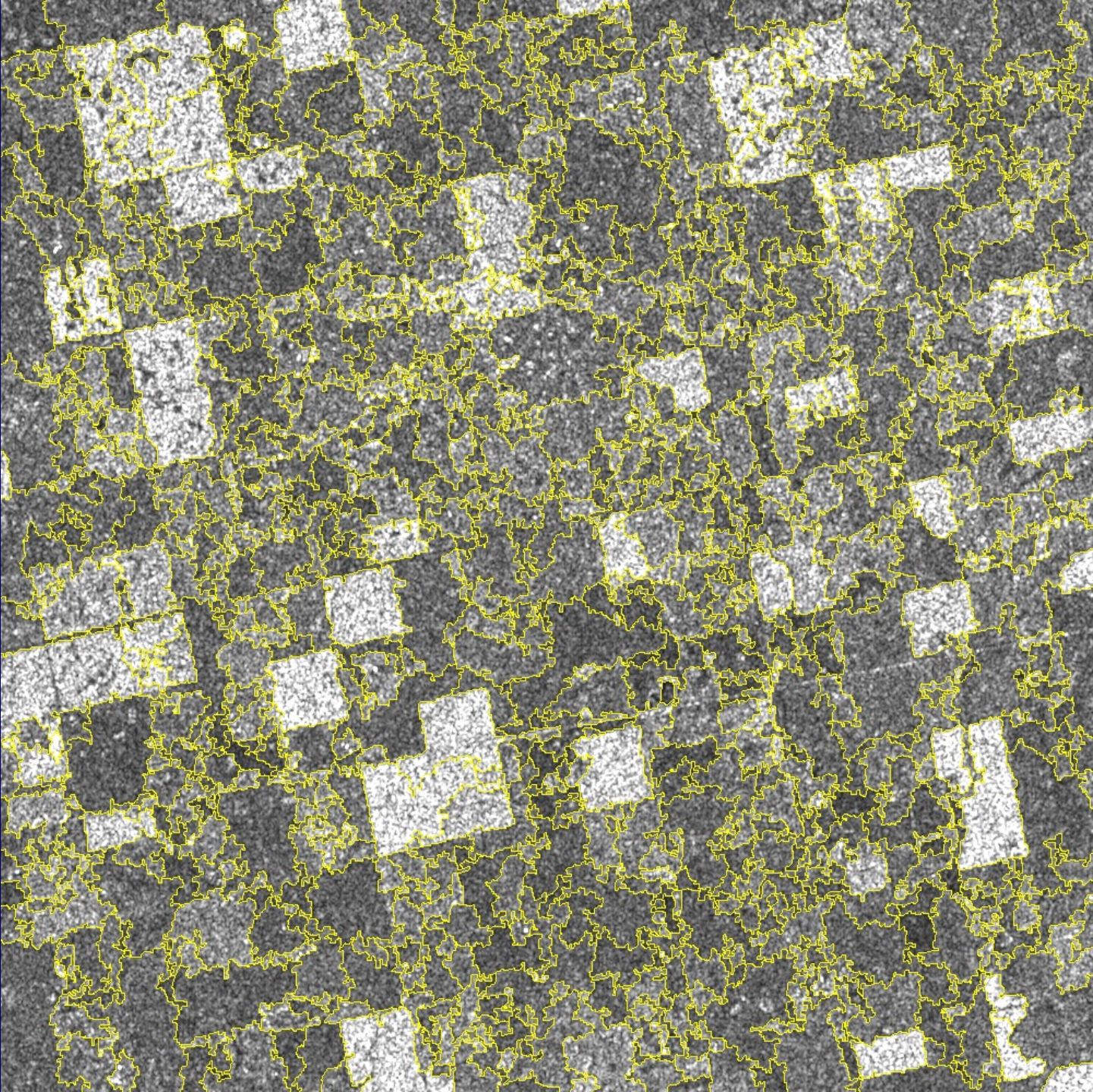
**1000x1000 SAR image**



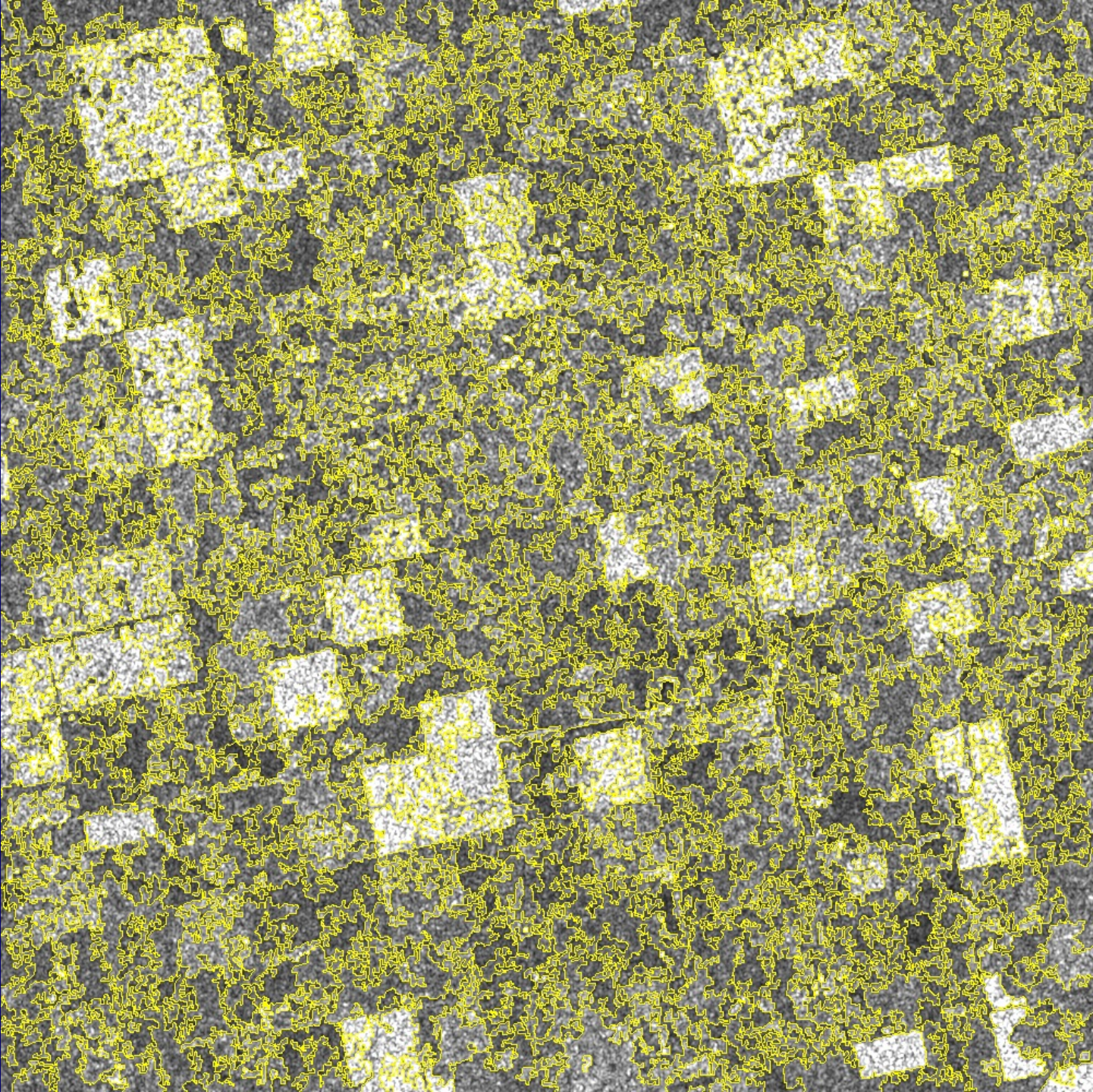
**1000 segments**



**1000 segments**



**1000 segments**



**1000 segments**

