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Segmentation of Polarimetric SAR images composed of textured and non-textured fields

Jean-Marie Beaulieu
Computer Science Department
Laval University

Ridha Touzi
Canada Centre for Remote Sensing
Natural Resources Canada

Segmentation of Polarimetric SAR images composed of textured and non-textured fields

- Segmentation by hypothesis testing
- Maximum likelihood approximation
- Segmentation of polarimetric images
- Evaluation of segmentation
- Texture extraction for image approximation

MULTILOOK IMAGE

For *L*-look image, a pixel *k* should be represented by its *L*-look covariance matrix, Z_k

Z_k follows a complex Wishart distribution

$$p(Z_k \mid \Sigma) = \frac{L^{3L} \left|Z_k\right|^{L-3} \quad \exp\left\{-L\, tr\left(\Sigma^{-1} Z_k\right)\right\}}{\pi^3\ \Gamma(L)\Gamma(L-1)\Gamma(L-2)\left|\Sigma\right|^L}$$

SEGMENTATION BY HYPOTHESIS TESTING

Test the similarity of segment covariances $C_i = C_j = C$

- merge segment with same covariance

Use the difference of determinant logarithms as a test statistic

$$C_{i,j} = K\left\{(n_{si} + n_{sj})\ln\left|C_{si\cup sj}\right| - n_{si}\ln\left|C_{si}\right| - n_{sj}\ln\left|C_{sj}\right|\right\}$$

With the scaling factor *K*, the statistic is approximately distributed as a chi-squared variable as n_{si} and n_{sj} become large.

False Alarm Rate (FAR) thresholding

Distribution of $C_{i,j}$ → FAR threshold

Design decision processes with constant FAR

Segmentation → compare two segments

Classification → compare one pixel with one class

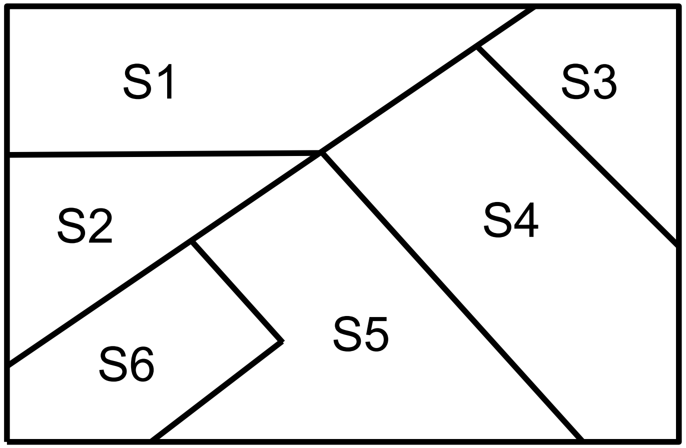
Local decision ←→ Global segmentation result

Sequence of tests

SEGMENTATION AS MAXIMUM LIKELIHOOD APPROXIMATION

1) need a partition of the image

$$P = \{s_k\}, \quad s_k = \{i\} \subset I$$



2) need statistical parameters

$$\theta = \{\theta_s\}, \quad s \in P$$

3) need an image probability model

$$p(x_i \mid \theta_s)$$

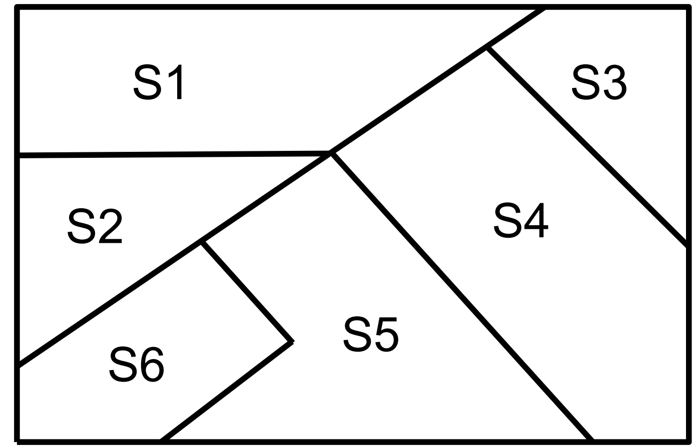
x_i are conditionally independent

Given an image $X = \{x_i\}, \quad i \in I$

the likelihood of $\theta = \{\theta_s\}, \quad P$

is $L(\theta, P \mid X) = p(X \mid \theta, P)$

$$L(\theta, P \mid X) = \prod_{i \in I} p(x_i \mid \theta_{s(i)})\Big|_P$$

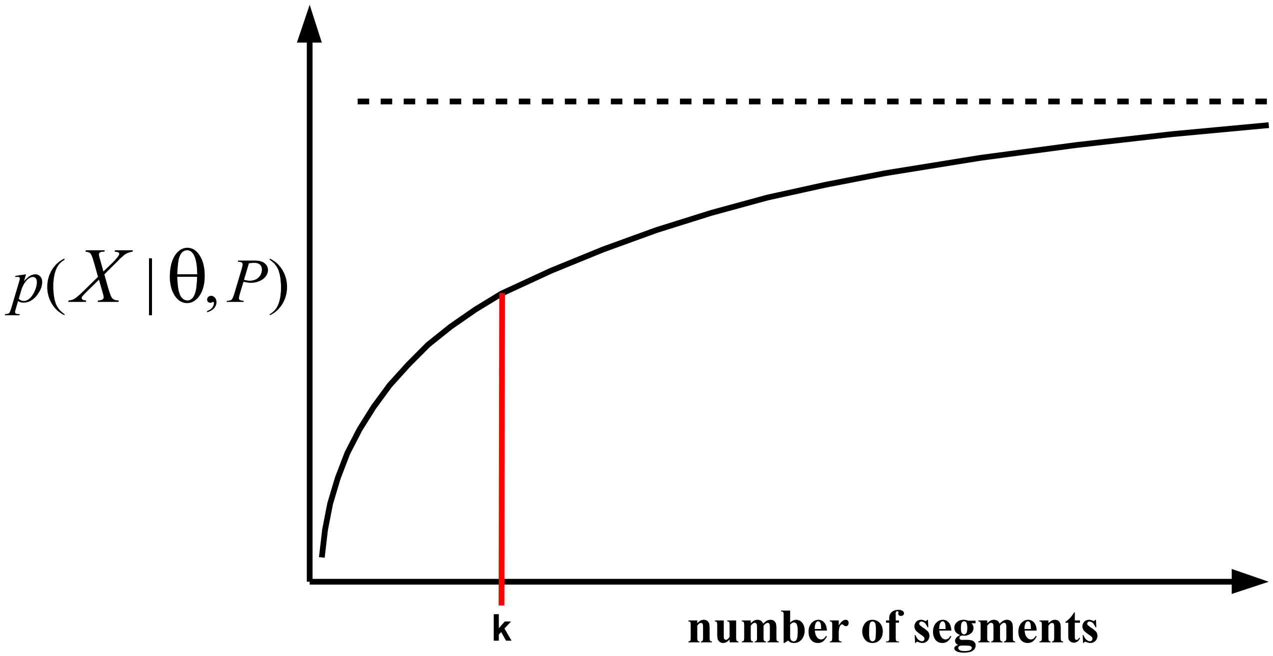


The segmentation problem is to find the partition that maximizes the likelihood.

Global search – too many possible partitions.

θ_s **is derived from statistics calculated over a segment *s*.**

The maximum likelihood increases with the number of segments

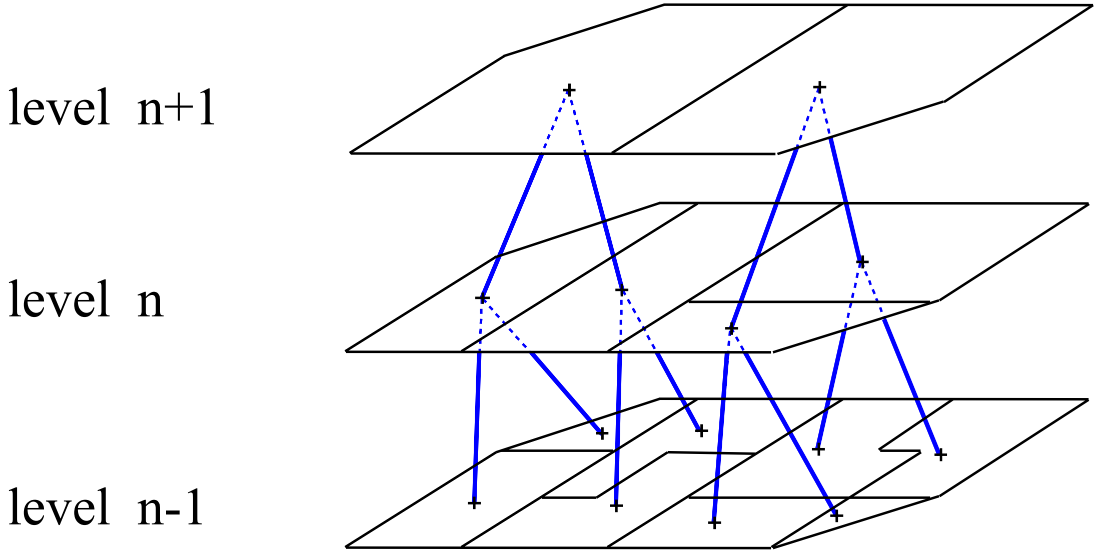


Can't find the optimum partition with k segments, P_k
Too many, except for P_1 and P_{nxn}.

Hierarchical segmentation
→ get P_k from P_{k+1} by merging 2 segments.

HIERARCHICAL SEGMENTATION

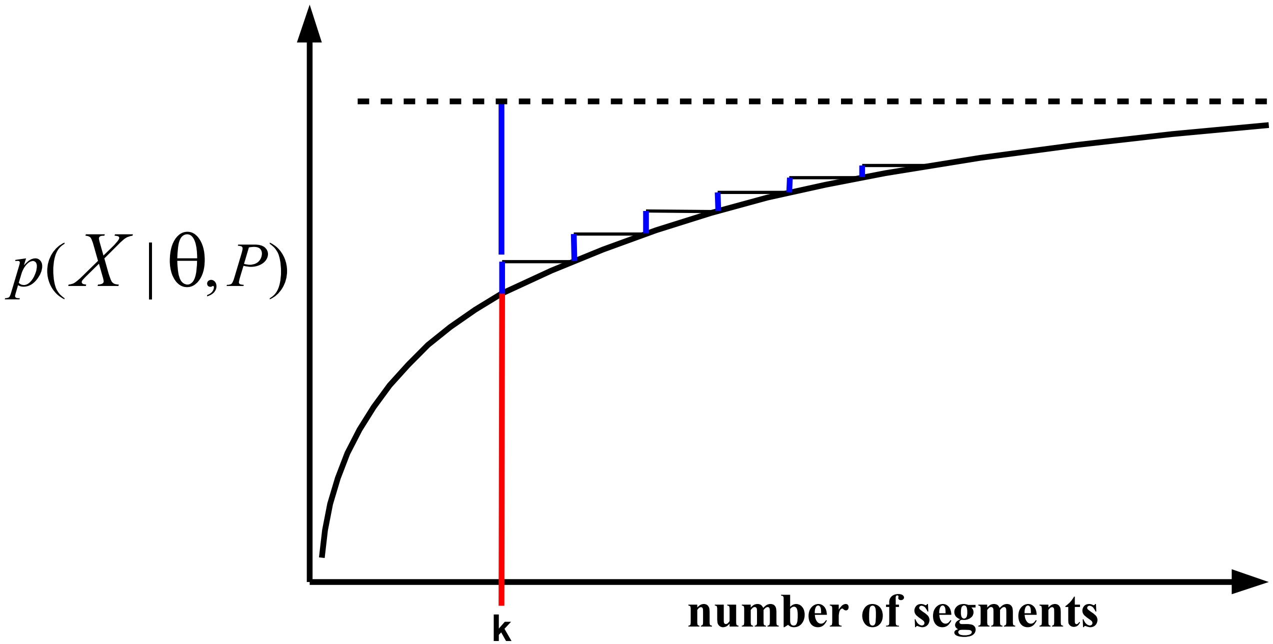
A hierarchical segmentation begins with an initial partition P^0 (with N segments) and then sequentially merges these segments.



Segment tree

Merging criterion:

merge the 2 segments producing the smallest decrease of the maximum likelihood (stepwise optimization)



Sub-optimum within hierarchical merging framework.

Log likelihood form

$$\ln\left(L(\theta, P \mid X)\right) = \ln\left(\prod_{i \in I} p(x_i \mid \theta_{s(i)})\right) = \sum_{i \in I} \ln\left(p(x_i \mid \theta_{s(i)})\right)$$

Summation inside region

$$LLF(P) = \sum_{S \in P} \sum_{i \in S} \ln\left(p(x_i \mid \theta_S)\right) = \sum_{S \in P} MLL(S)$$

Criterion → cost of merging 2 segments

$$\Delta = MLL(S_i) + MLL(S_j) - MLL(S_i \cup S_j)$$

$$\Delta = \sum_{x \in S_i} \ln\left(p(x \mid \theta_{S_i})\right) + \sum_{x \in S_j} \ln\left(p(x \mid \theta_{S_j})\right) - \sum_{x \in S_i \cup S_j} \ln\left(p(x \mid \theta_{S_i \cup S_j})\right)$$

minimize $|\Delta|$

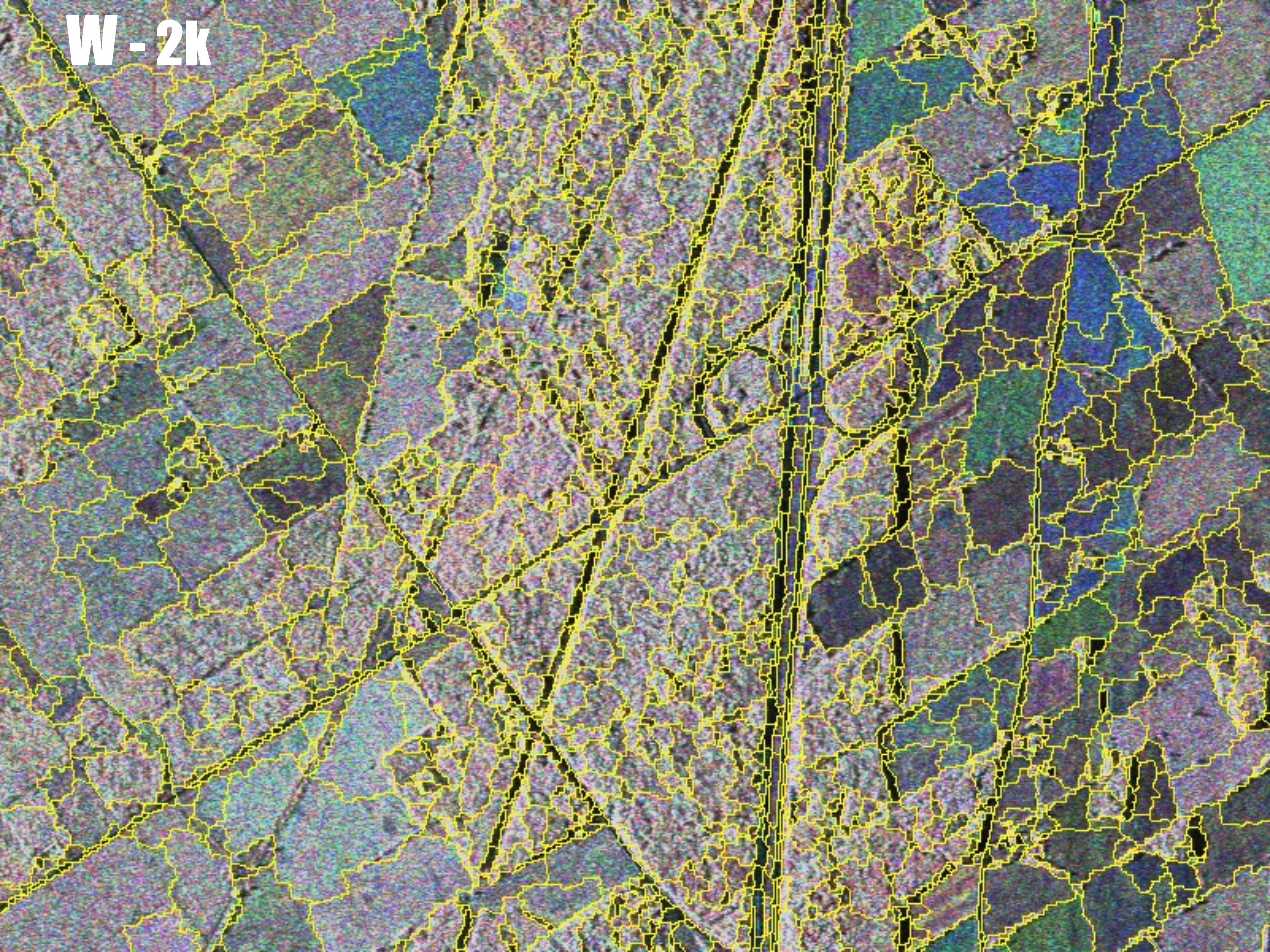
HOMOGENEOUS IMAGE

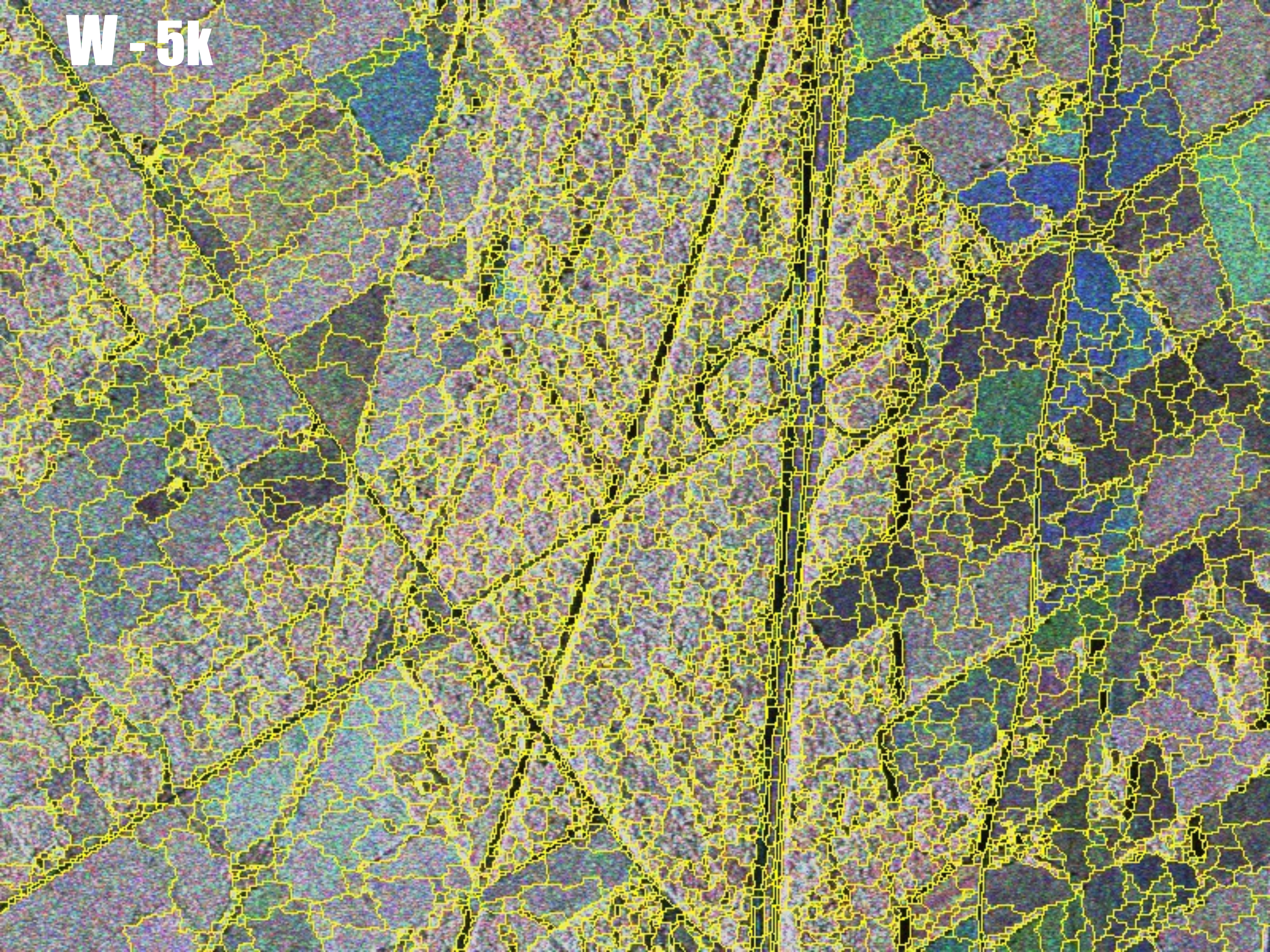
The stepwise criterion is

$$C_{i,j} = (n_{si} + n_{sj}) \ln\left|C_{si \cup sj}\right| - n_{si} \ln\left|C_{si}\right| - n_{sj} \ln\left|C_{sj}\right|$$

This is equivalent to the hypothesis testing criterion.

Hierarchical segmentation by stepwise optimisation.


W - 2k


W - 5k

TEXTURED IMAGE

Assume that a texture value μ modifies the covariance matrix

$$\boldsymbol{Z}_{\mathbf{k}} = \mu_{\mathbf{k}} \boldsymbol{Z}_{\mathbf{k\text{-homogeneous}}}$$

$\boldsymbol{Z}_{\mathbf{k}}$ **follows a K distribution**

$$p(Z_k \mid \alpha, \Sigma) = \frac{(\alpha L)^{(3L+\alpha)/2}\ 2\left|Z_k\right|^{L-3} \left(tr\left(\Sigma^{-1} Z_k\right)\right)^{(\alpha-3L)/2}}{\pi^3\ \Gamma(L)\Gamma(L-1)\Gamma(L-2)\ \Gamma(\alpha)\left|\Sigma\right|^{L}} K_{3L-\alpha}\left\{2\sqrt{\alpha L\ tr\left(\Sigma^{-1} Z_k\right)}\right\}$$

The maximum log likelihood for one segment is

$$MLL(S) \simeq n\frac{3L+\alpha}{2}\ln(\alpha L) - n\ln(\Gamma(\alpha)) - nL\ln(|\Sigma|)$$
$$+\frac{\alpha-3L}{2}\sum_{k\in S}\ln\left(tr\left(\Sigma^{-1}Z_k\right)\right)$$
$$+\sum_{k\in S}K_{3L-\alpha}\left\{2\sqrt{\alpha L\, tr\left(\Sigma^{-1}Z_k\right)}\right\}$$

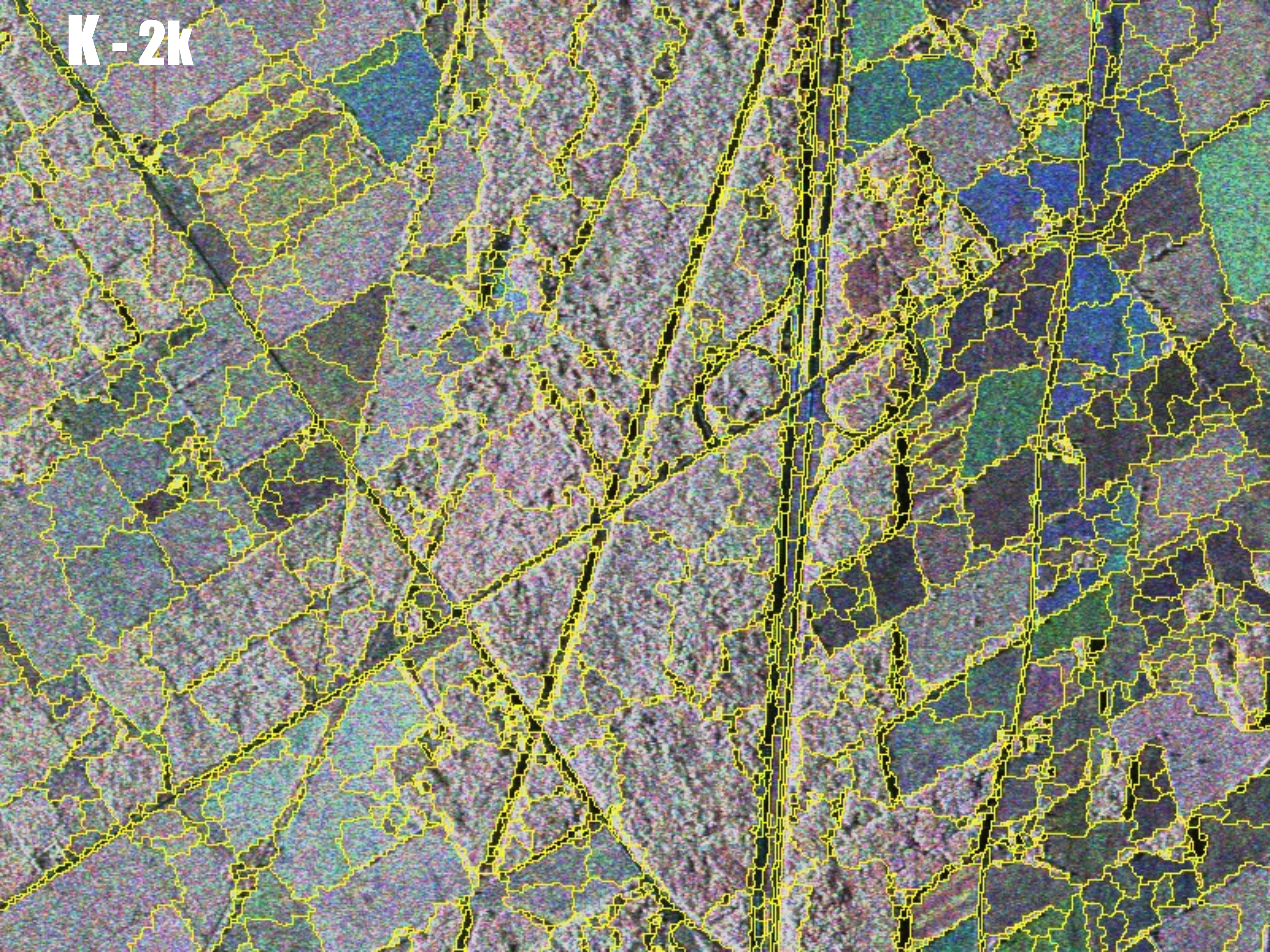
Best α and Σ → Iteration (gradient descent)

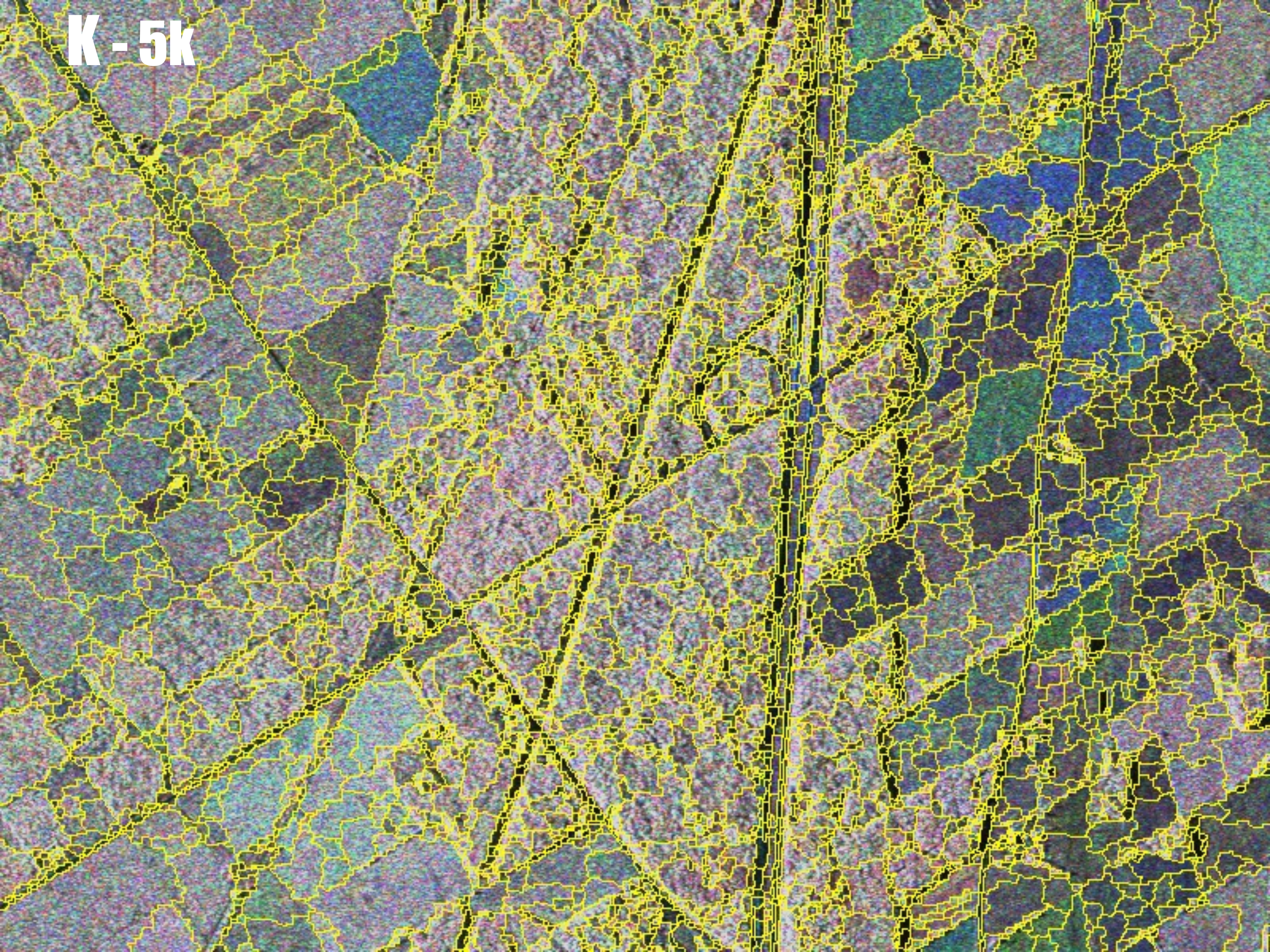
Approximation

Σ = segment covariance matrix

$\alpha = 1/(CV_R)^2$ → Method of Moments

$$C_{i,j} = MLL(S_i) + MLL(S_j) - MLL(S_i \cup S_j)$$

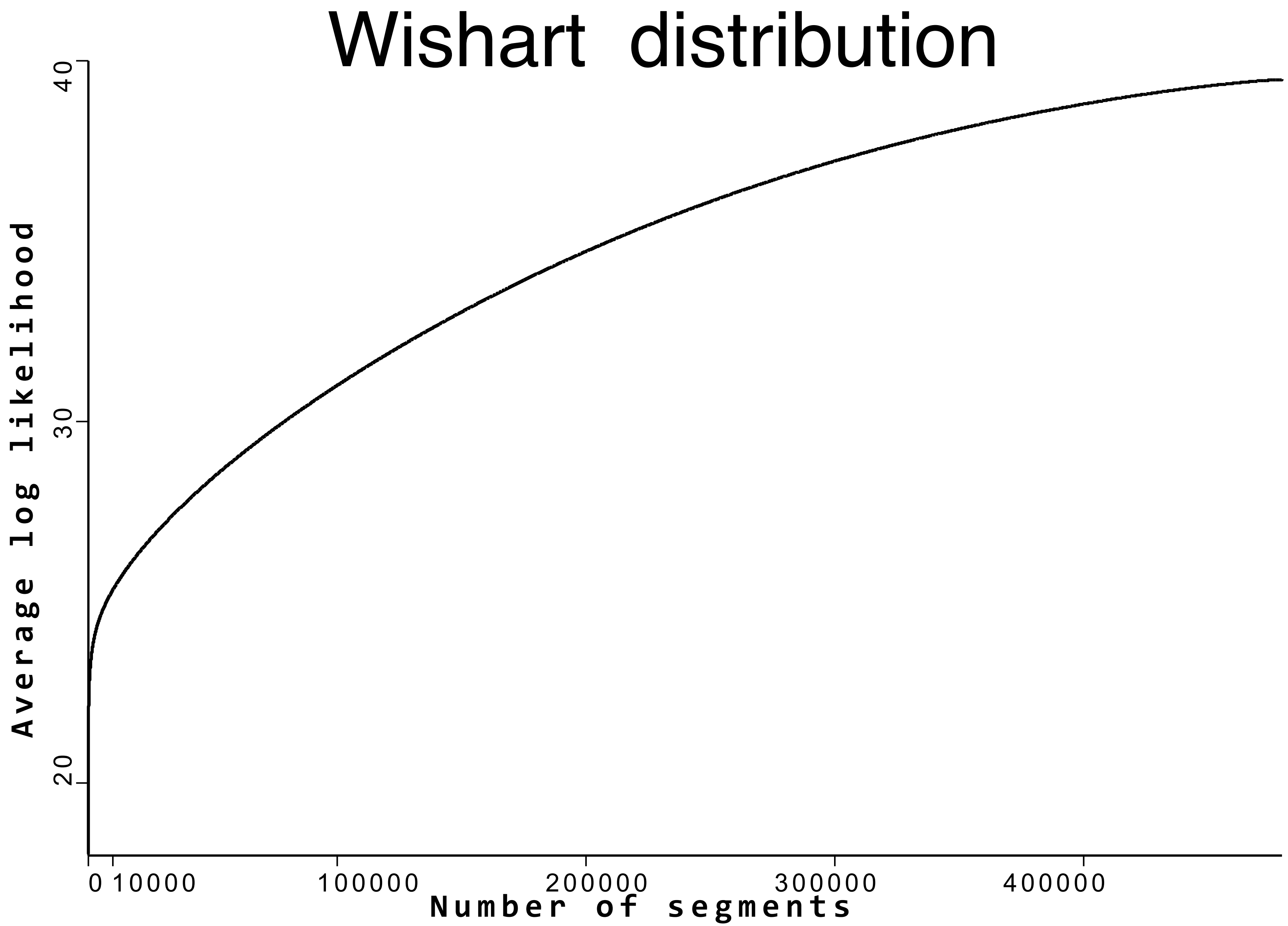

K - 2k


K - 5k

EVALUATION OF SEGMENTATION

Mesure the likelihood value of the partition

$$LLF(P) = \ln\left(L(\Theta_P, P \mid X)\right) = \sum_{S \in P} MLL(S)$$

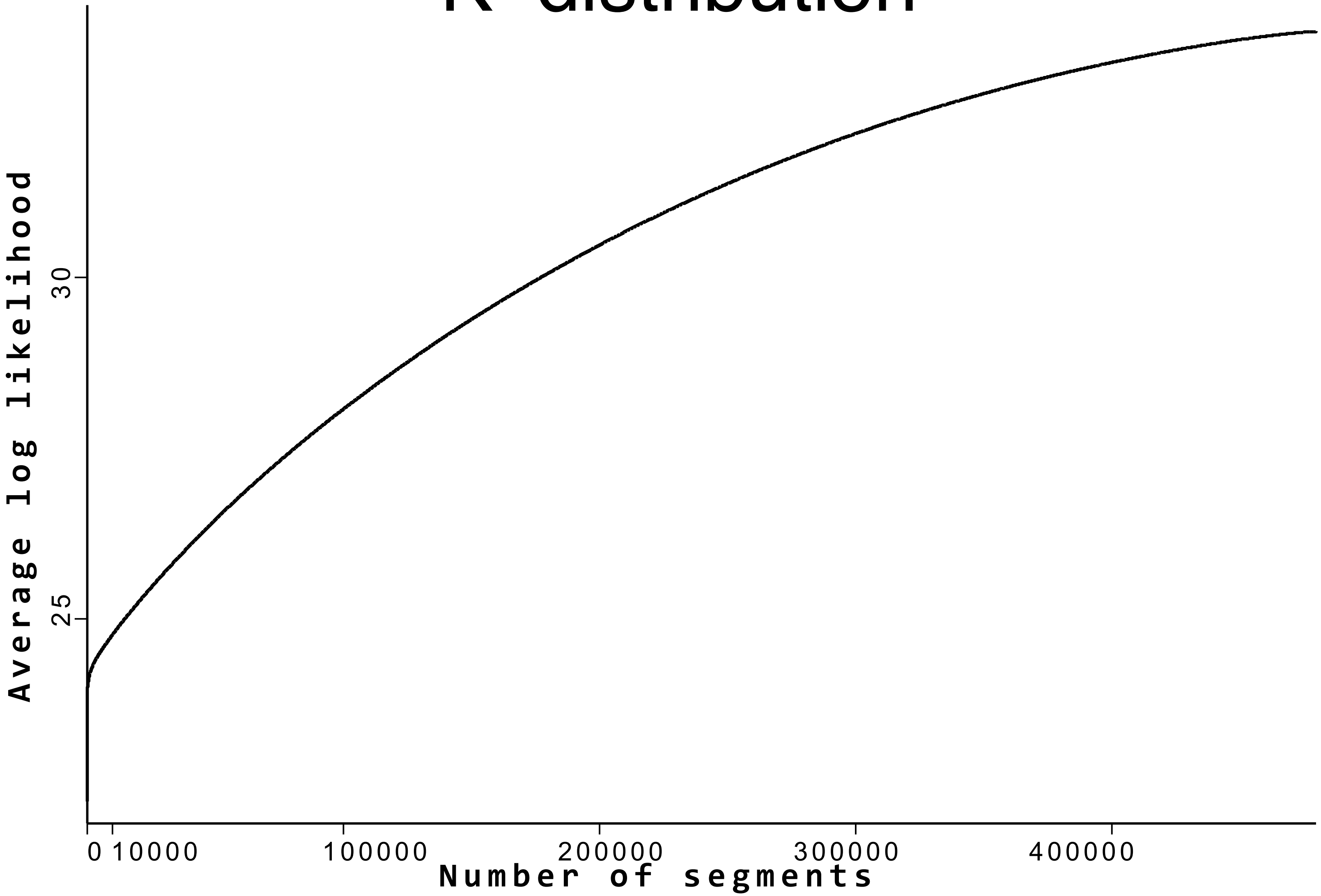


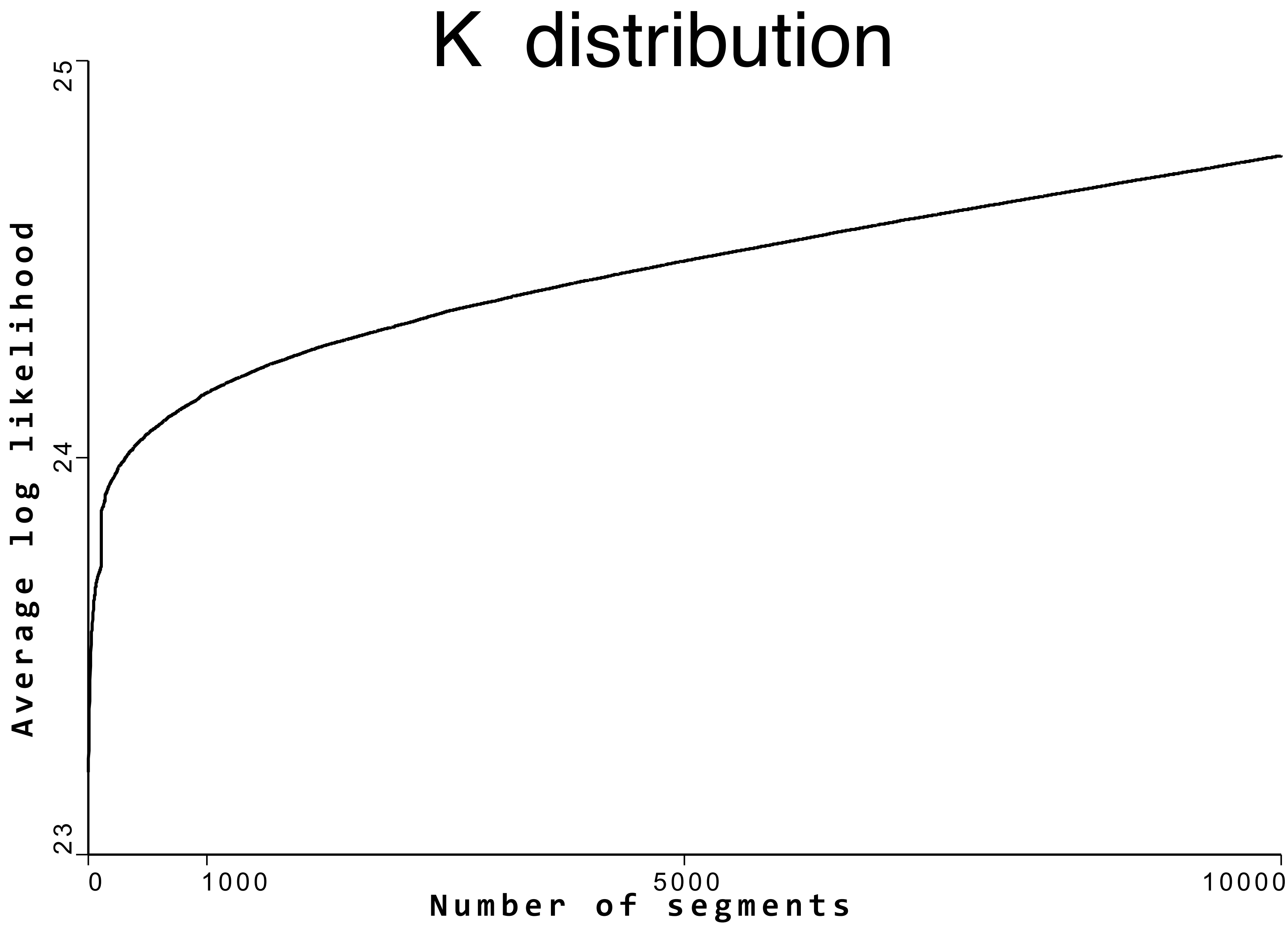
Wishart distribution
Average log likelihood
40
30
20
0 10000
100000
200000
300000
400000
Number of segments

Wishart distribution

Wishart distribution

K distribution




K distribution
Average log likelihood
Number of segments
25
24
23
0
1000
5000
10000

EVALUATION OF SEGMENTATION

One channel SAR image → ratio image $\frac{I}{I_S}$ **(noise image)**

Wishart distribution ? → determinant ratio image

Pixel log likelihood

$$LL(Z_k) = (L-3)\ln|Z_k| - L\ln|C_S| - L\, tr\left(C_S^{-1} Z_k\right) - \ln\left(Q(L)\right)$$

Avera over the image

$$\sum LL(Z_k) \propto \sum \ln\left(\frac{|Z_k|}{|C_S|}\right)$$

Log determinant ratio

EVALUATION OF SEGMENTATION

One channel SAR image → ratio image $\dfrac{I}{I_S}$ **(noise image)**

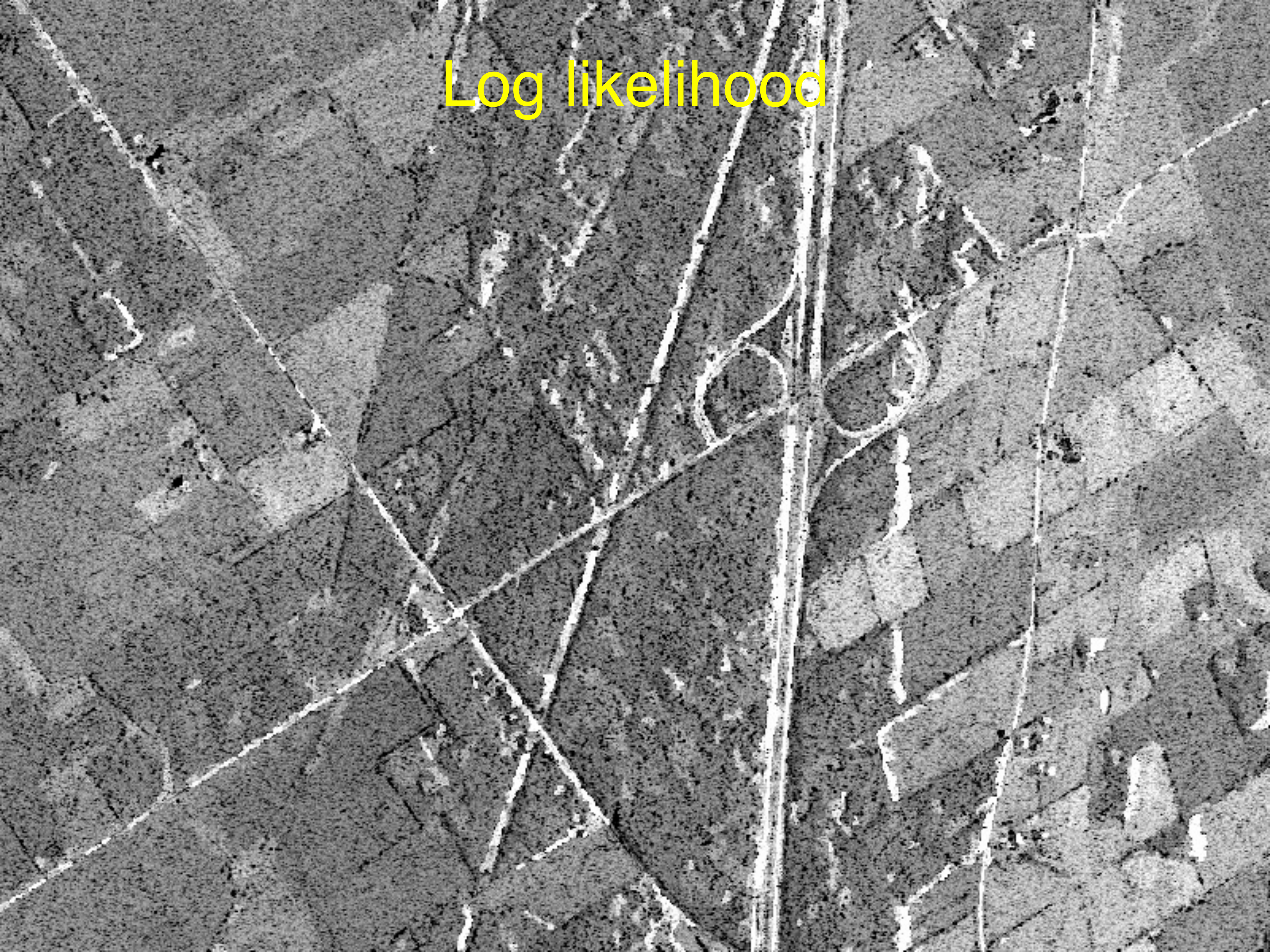
Wishart distribution ? → ~~determinant ratio image~~

Pixel log likelihood

$$LL(Z_k) = (L-3)\ln|Z_k| - L\ln|C_S| - L\,tr\left(C_S^{-1}Z_k\right) - \ln\left(Q(L)\right)$$

Avera over the image

$$\sum LL(Z_k) \propto \sum \ln\left(\frac{|Z_k|}{|C_S|}\right)$$


Log likelihood

EVALUATION OF SEGMENTATION

One channel SAR image → ratio image $\dfrac{I}{I_S}$ **(noise image)**

Wishart distribution ? → ~~**determinant ratio image**~~

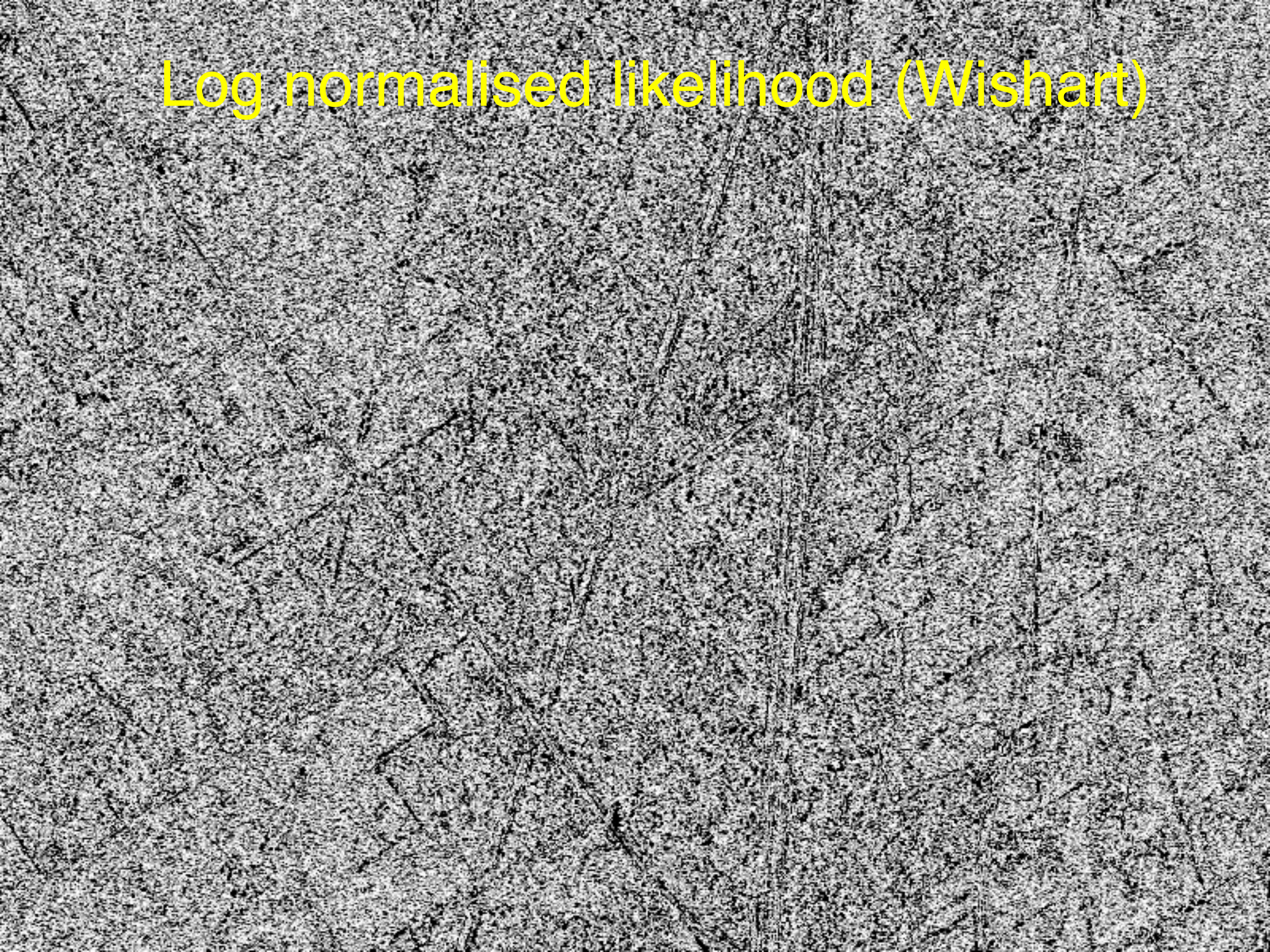
log normalised likelihood

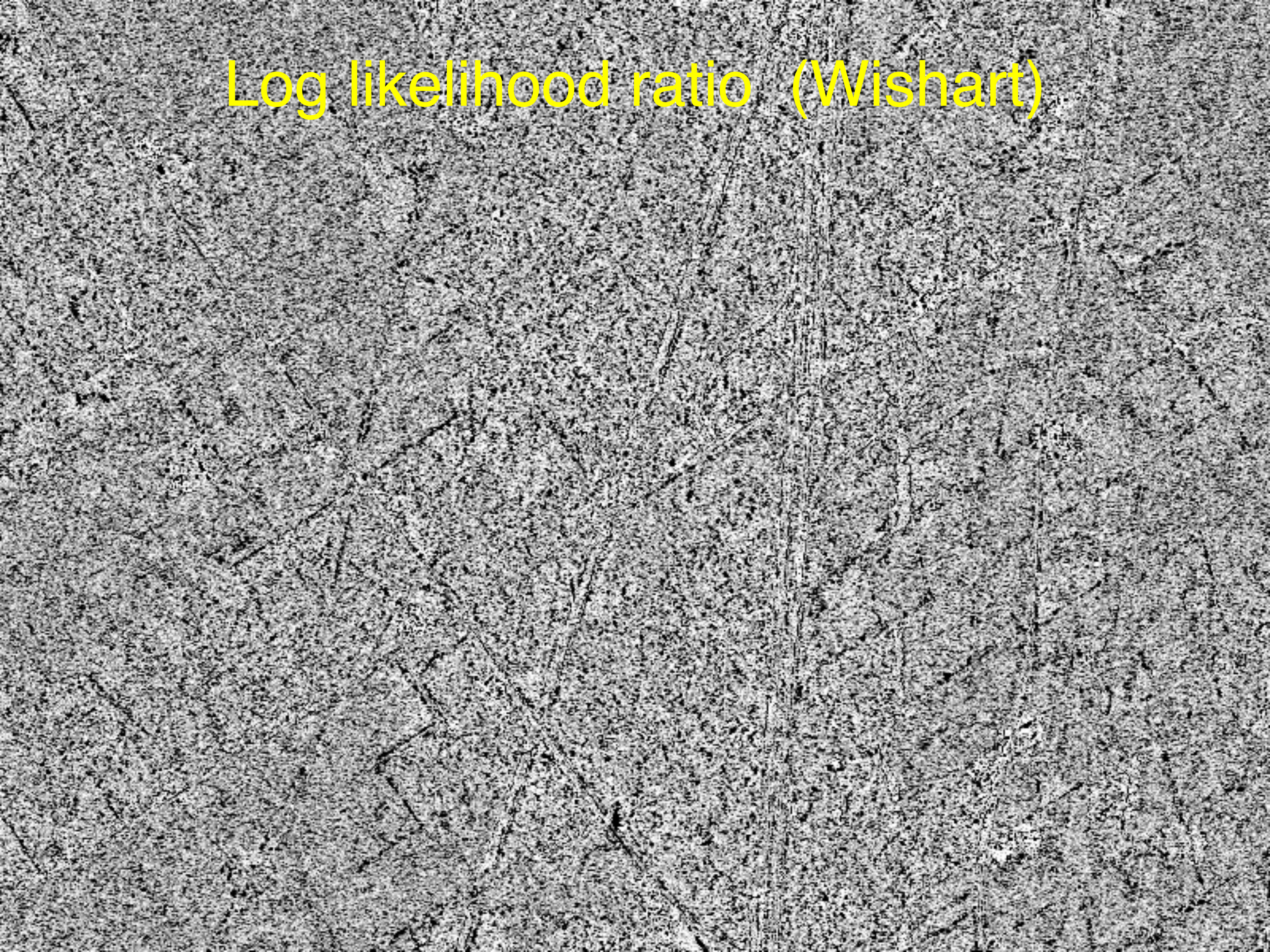
$$|C_S| = 1 \qquad LL(Z_k) = (L-3)\ln|Z_k| - L\ln|C_S| - L\,tr\left(C_S^{-1}Z_k\right) - \ln\left(Q(L)\right)$$

log likelihood ratio

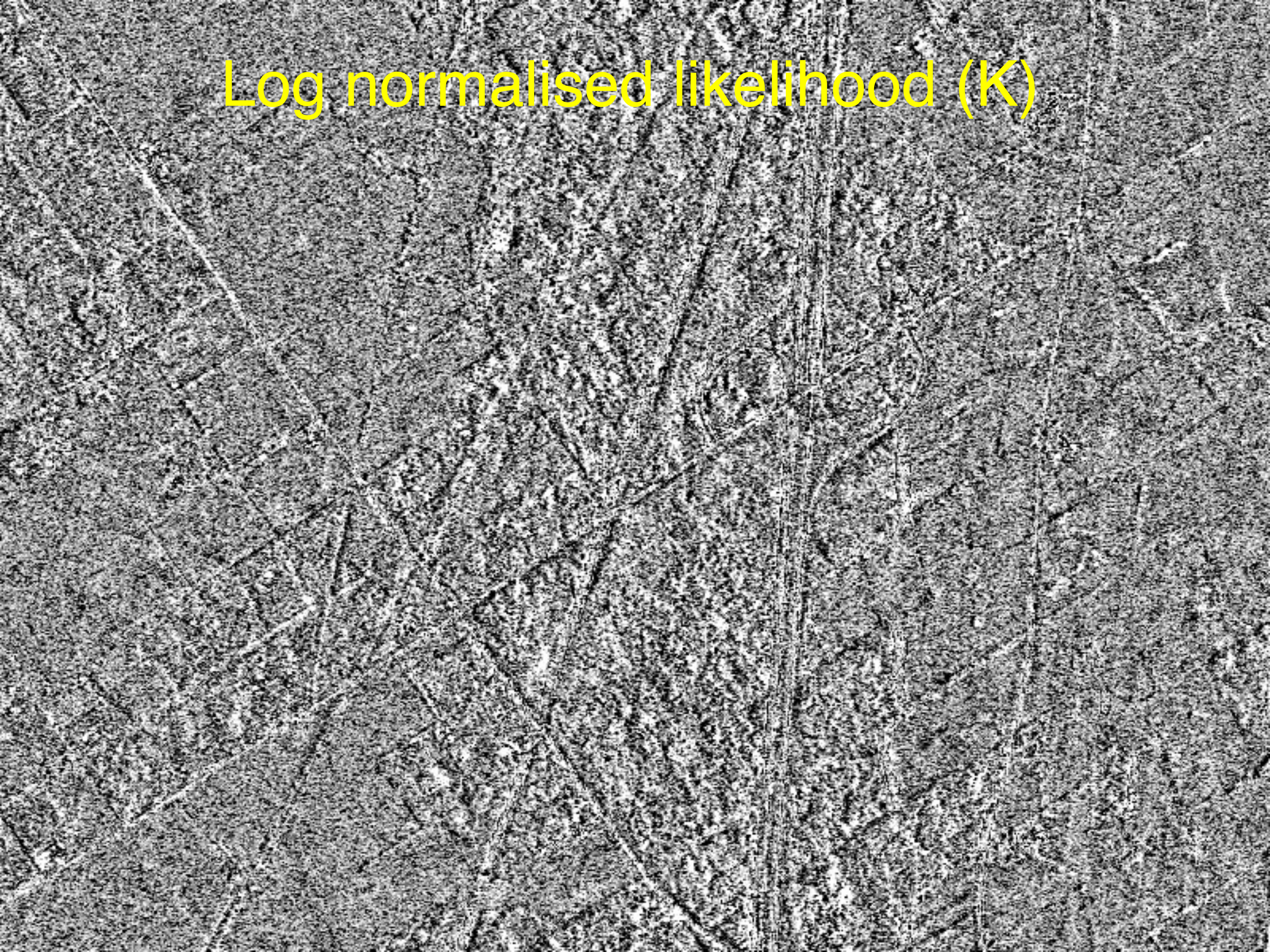
$$\log\left\{\frac{pixel \;\; likelihood}{average \;\; segment \;\; likelihood}\right\}$$

Log normalised likelihood (Wishart)




Log likelihood ratio (Wishart)

Log normalised likelihood (K)



Log likelihood ratio (K)

TEXTURED IMAGE

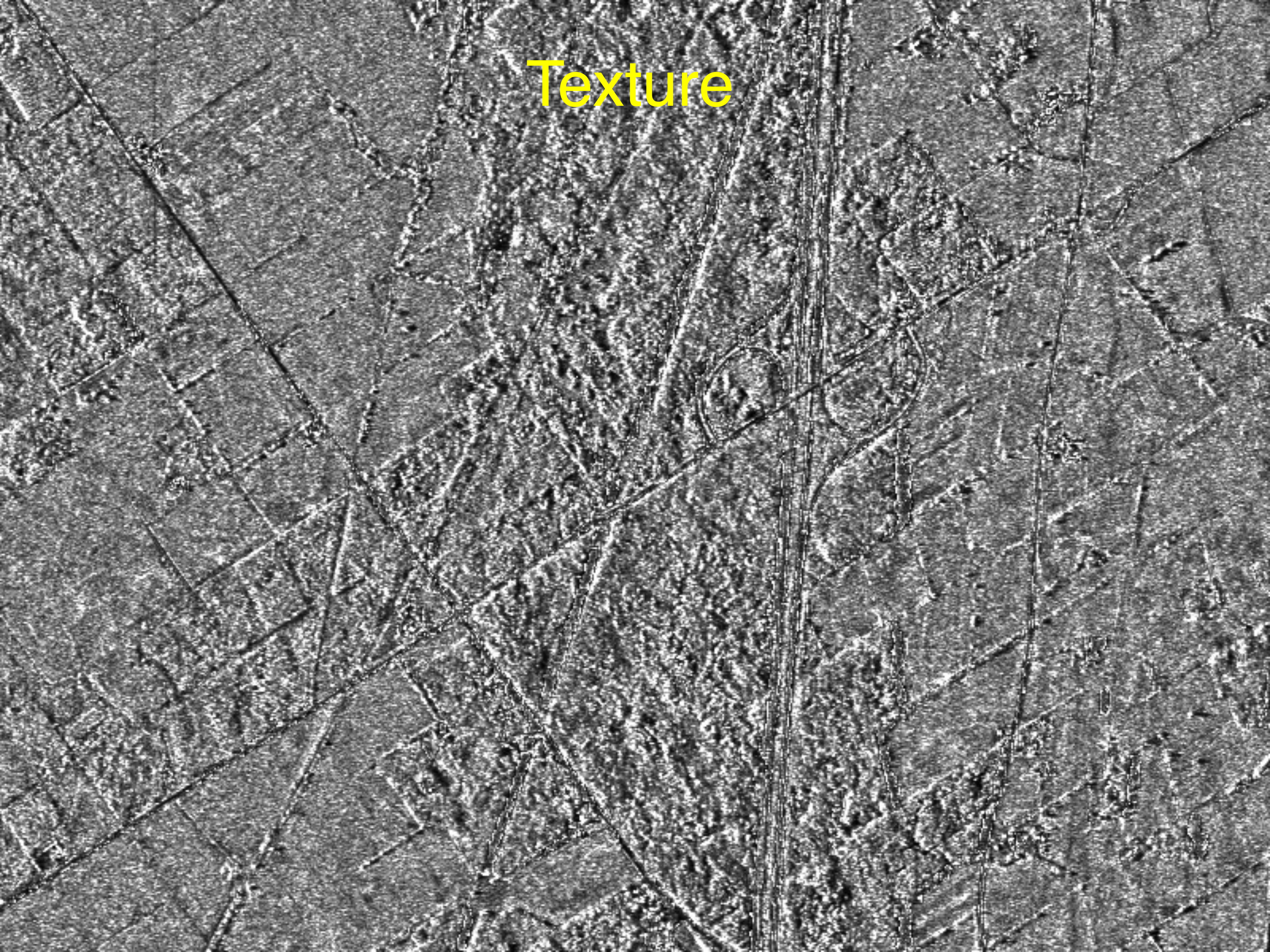
Separation of texture value μ and speckle covariance matrix

$$\boldsymbol{Z}_{\mathbf{k}} = \mu_{\mathbf{k}} \, \boldsymbol{Z}_{\mathbf{k\text{-}homogeneous}}$$

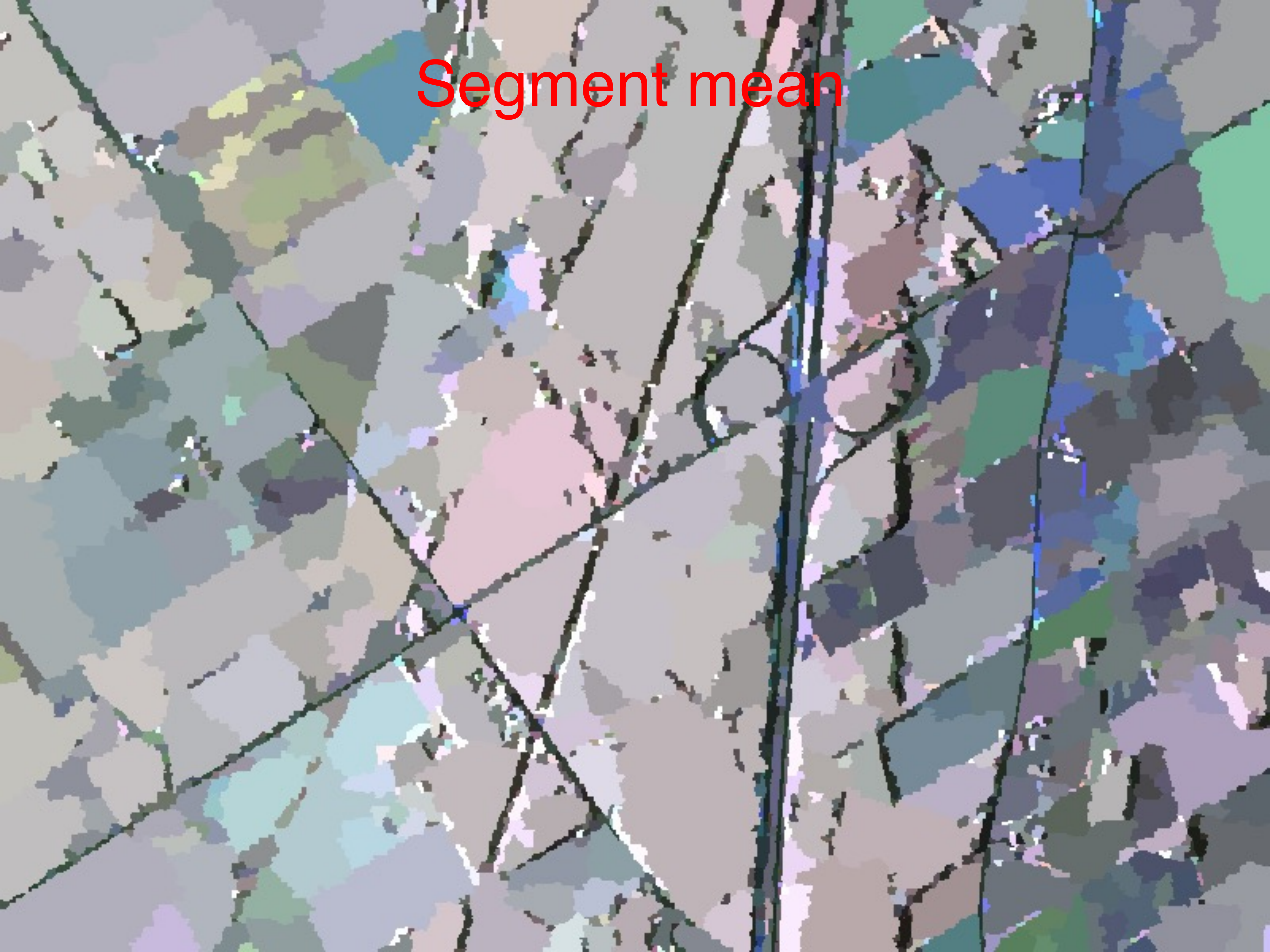
Iterative texture evaluation

$$\mu_k = \tfrac{1}{3} \, tr\left\{ C_S^{-1} Z_k \right\}$$

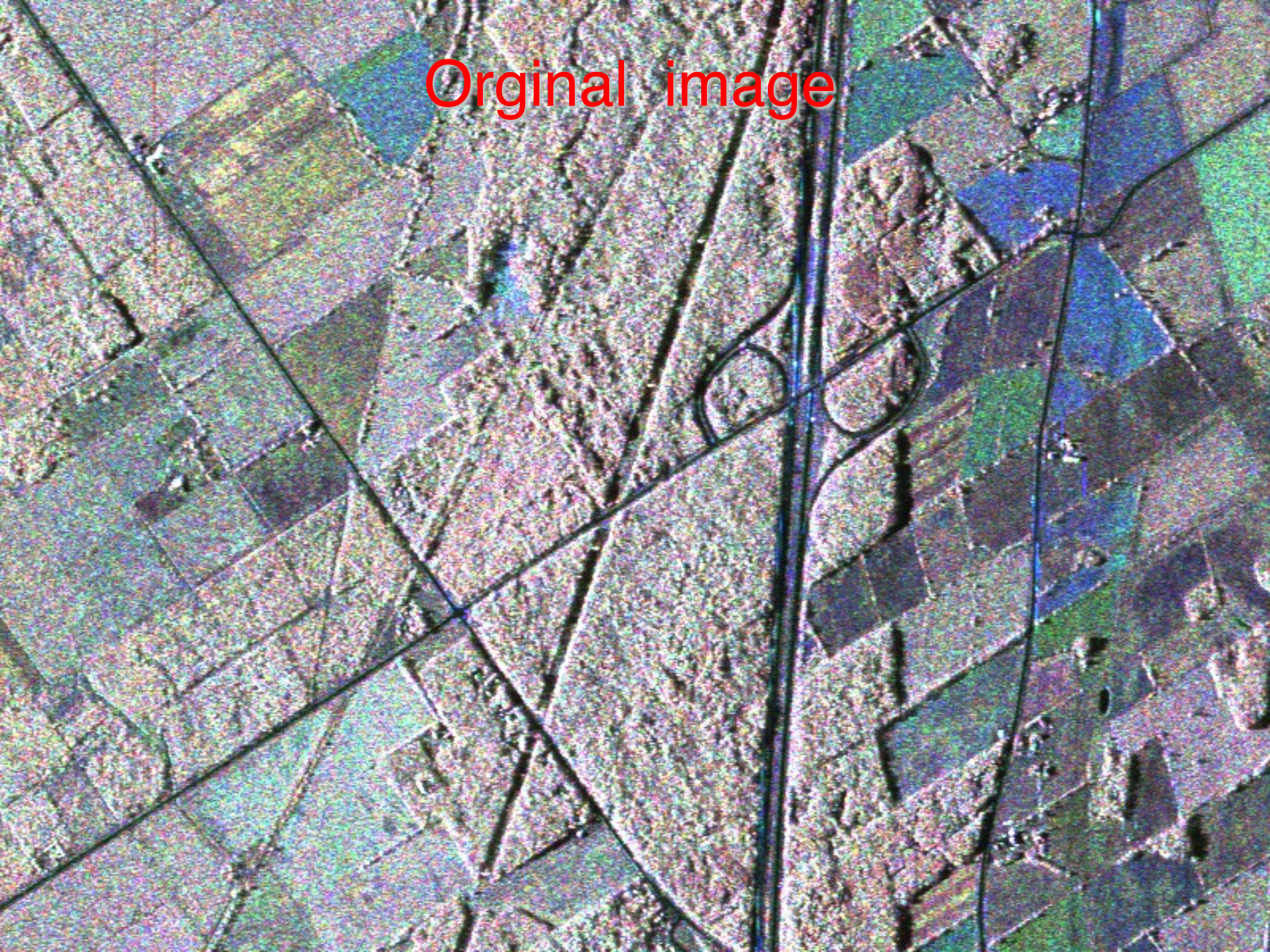
$$C_S = \tfrac{1}{N} \sum_{k \in S} \frac{1}{\mu_k} Z_k$$


Texture


Corrected covariance

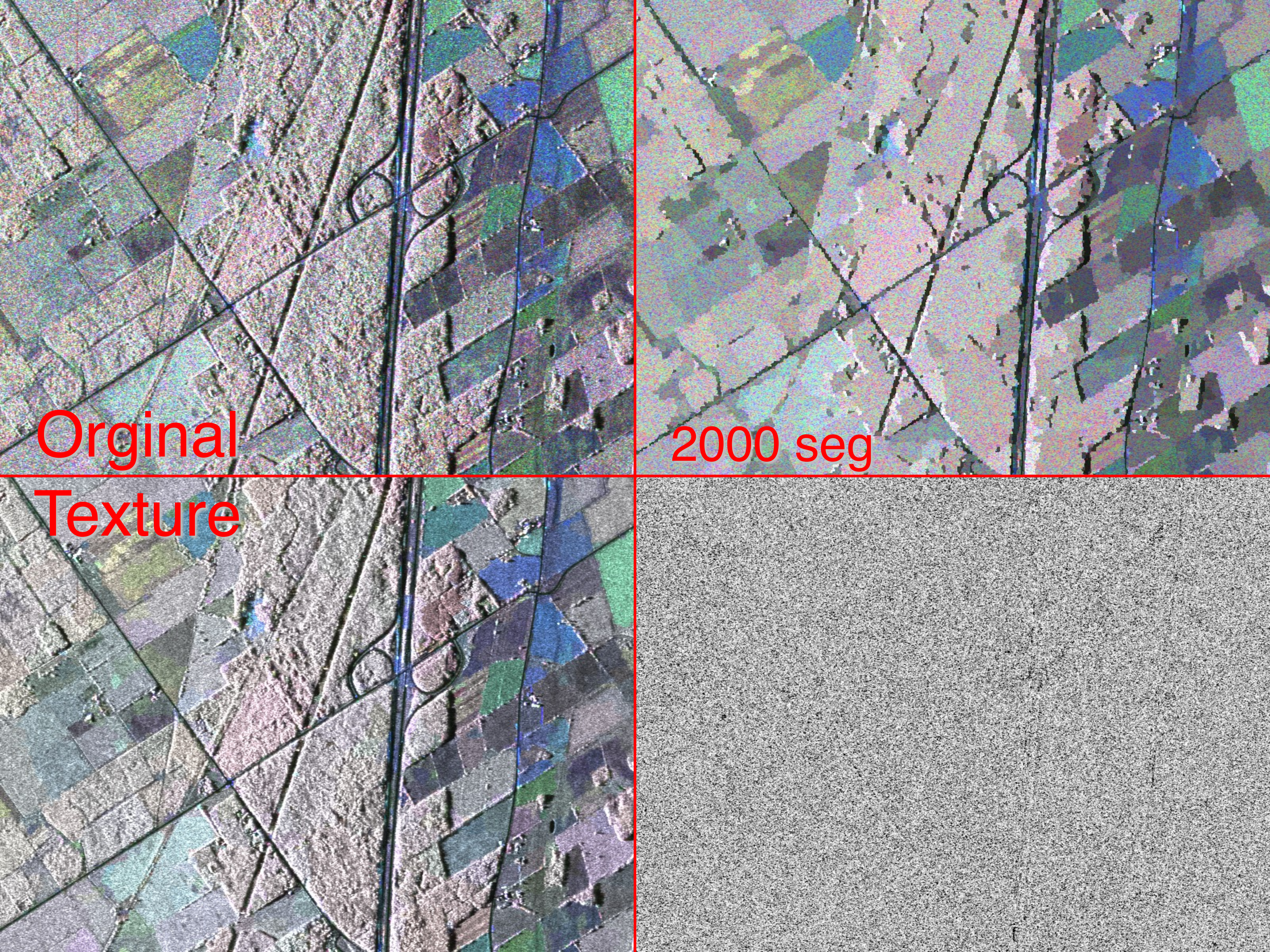

Segment mean

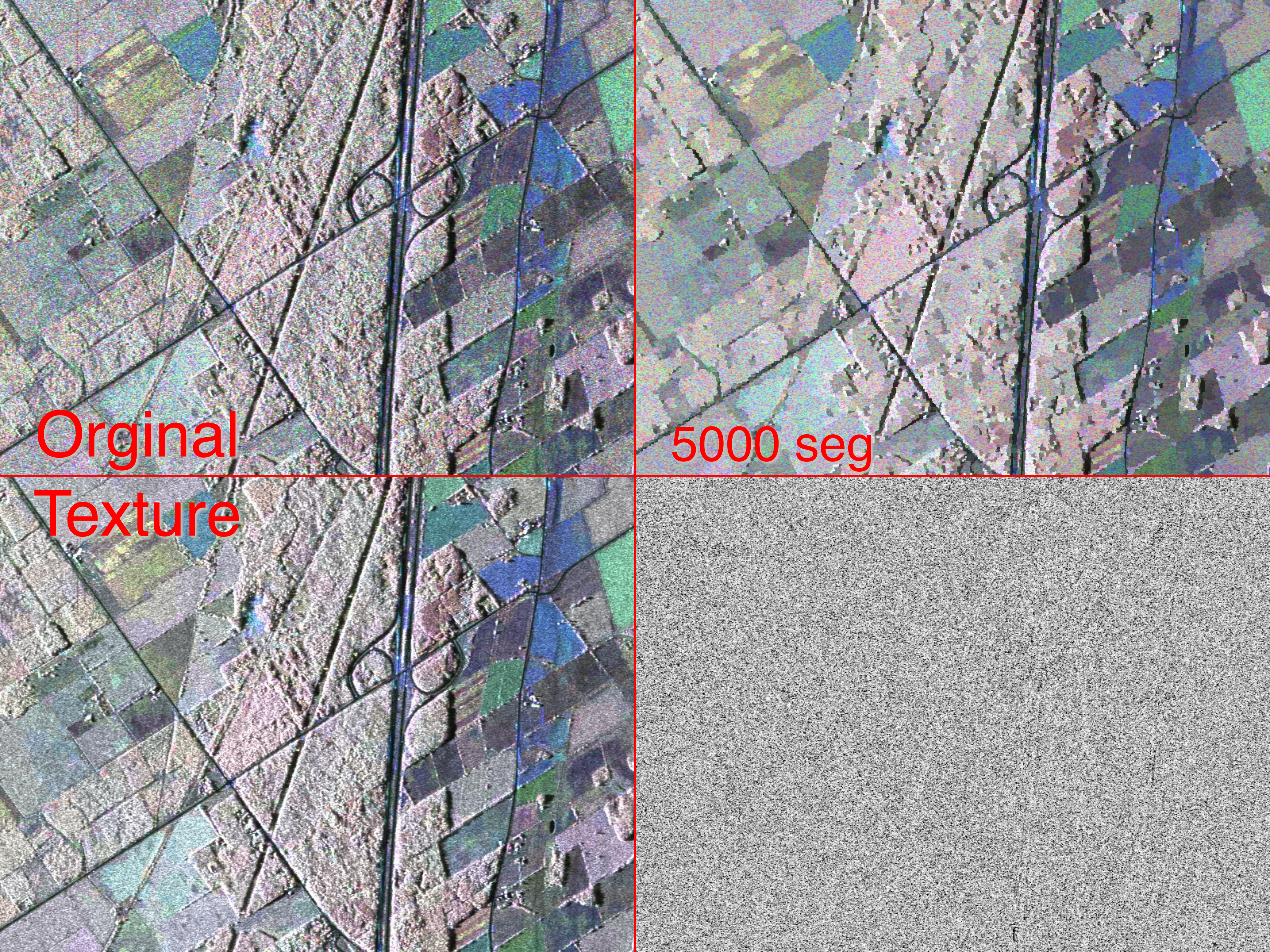

Texture x mean


Orginal image


Texture x mean (5000 segments)

Log normalised likelihood


Orginal
2000 seg
Texture


Orginal
5000 seg
Texture

CRITERION FOR SMALL SEGMENTS

The determinant |C| is null for small segments

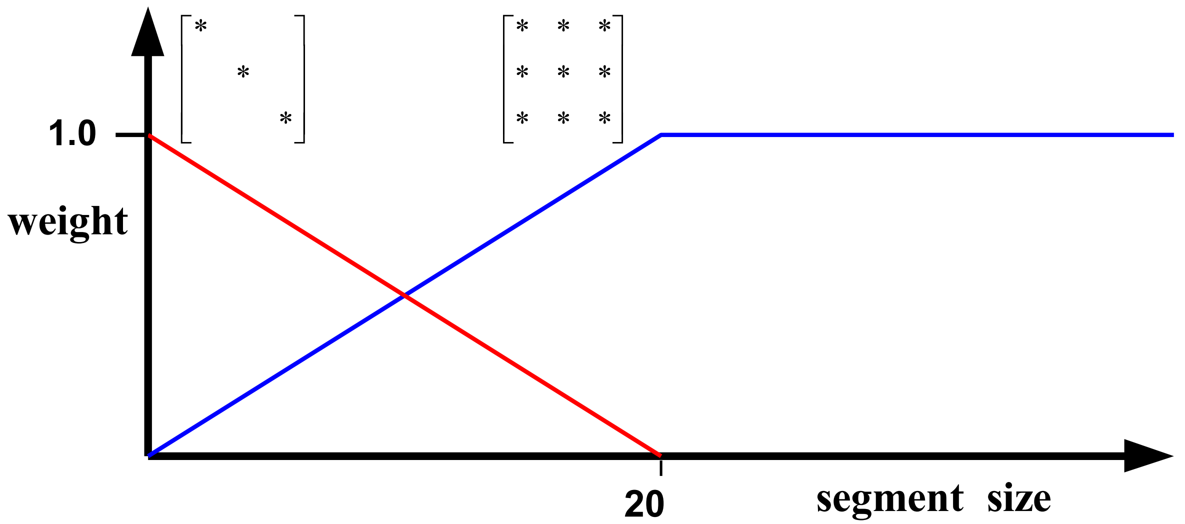
$$\boldsymbol{C} = \frac{1}{n}\begin{bmatrix} \sum hh\,hh^* & \sum hh\,hv^* & \sum hh\,vv^* \\ \sum hv\,hh^* & \sum hv\,hv^* & \sum hv\,vv^* \\ \sum vv\,hh^* & \sum vv\,hv^* & \sum vv\,vv^* \end{bmatrix}$$

Reduce covariance matrix model for small segments

$$\frac{1}{n}\begin{bmatrix} \sum hh\,hh^* & 0 & \sum hh\,vv^* \\ 0 & \sum hv\,hv^* & 0 \\ \sum vv\,hh^* & 0 & \sum vv\,vv^* \end{bmatrix}$$

$$\frac{1}{n}\begin{bmatrix} \sum hh\,hh^* & 0 & 0 \\ 0 & \sum hv\,hv^* & 0 \\ 0 & 0 & \sum vv\,vv^* \end{bmatrix}$$

Gradual transition between models

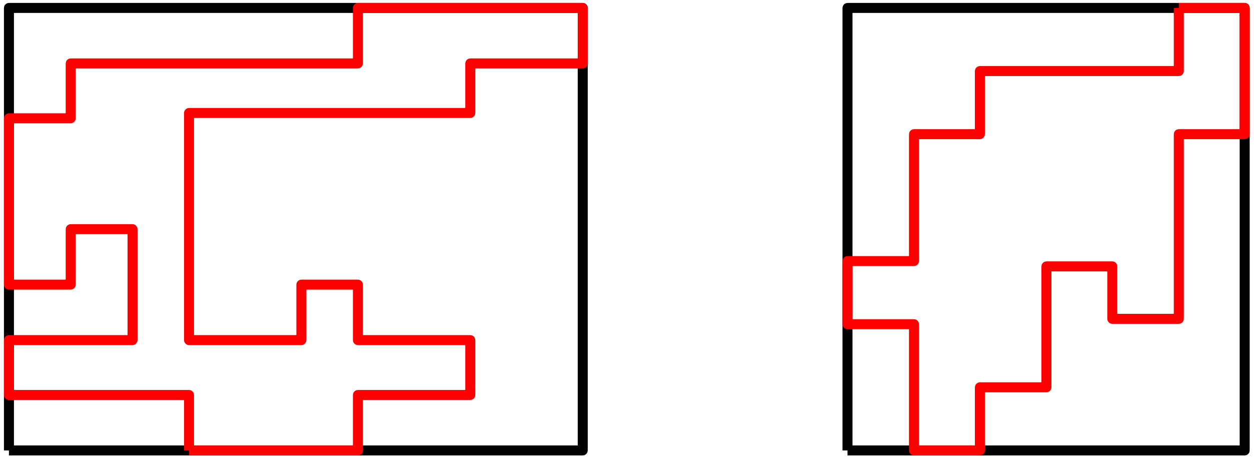


CONCLUSION

- Likelihood approximation produces good results
- Good polarimetric criteria for
  homogeneous and textured fields
- Texture separation is useful for evaluation

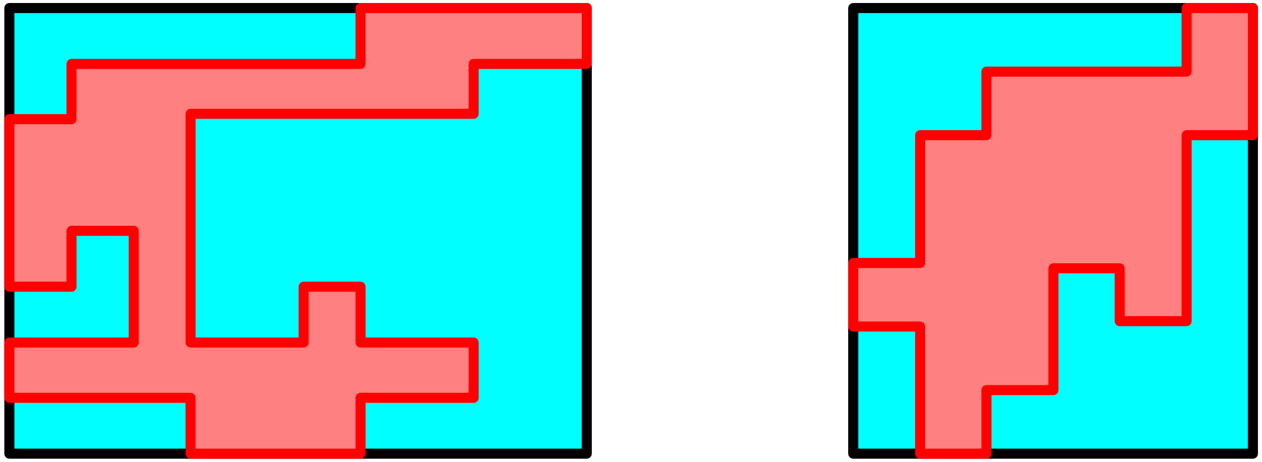
Bonding box – perimeter

$$Cp = \frac{perimeter\ of\ S_i \cup S_j}{perimeter\ of\ bonding\ box}$$



Bonding box – area

$$Ca = \frac{area\ of\ bonding\ box}{area\ of\ S_i \cup S_j}$$

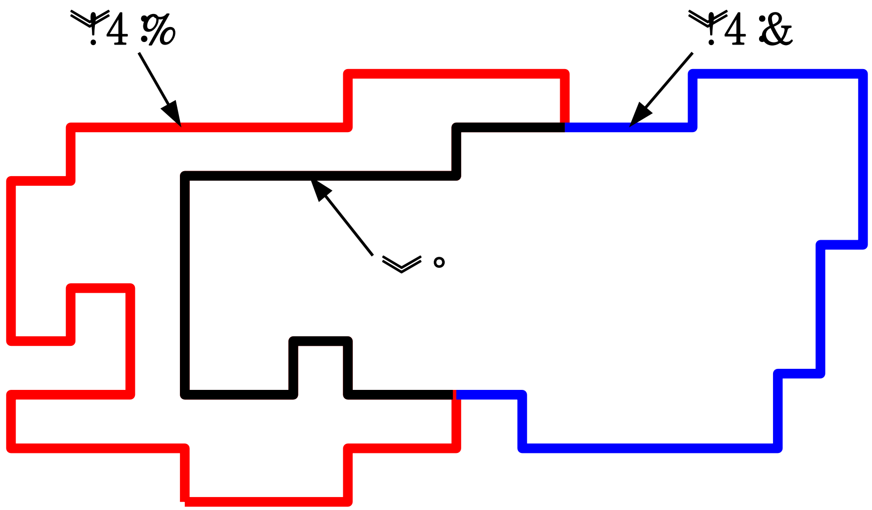


Contour length

$$Lc = \text{length of common part of contours}$$

$$Lex\,i = \text{length of exclusive part for } S_i$$

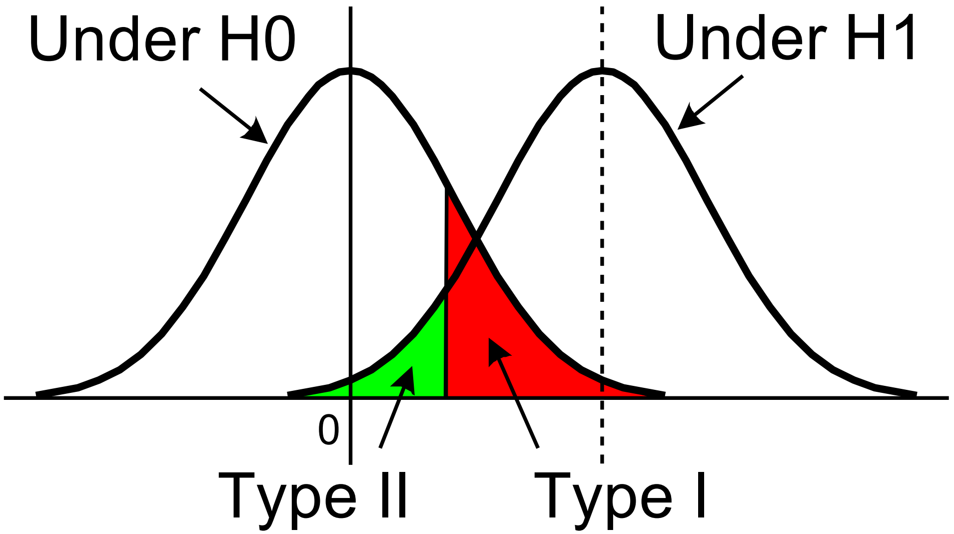
$$Cl = Min\left\{ \frac{Lex\,i}{Lc}, \quad \frac{Lex\,j}{Lc} \right\}$$



Segmentation by hypothesis testing

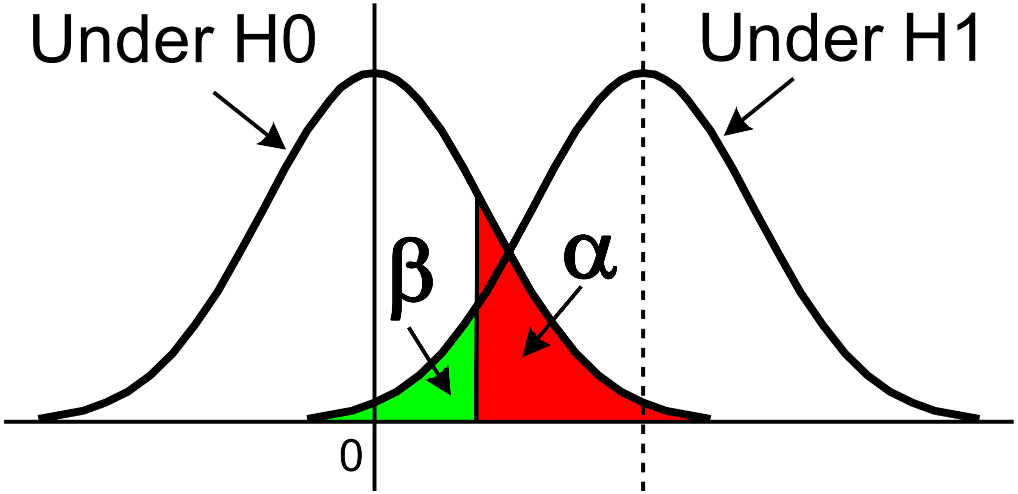
Two hypothesis
H0: segments are similar
H1: segments are different

Distributions of the statistic d under H0 and H1



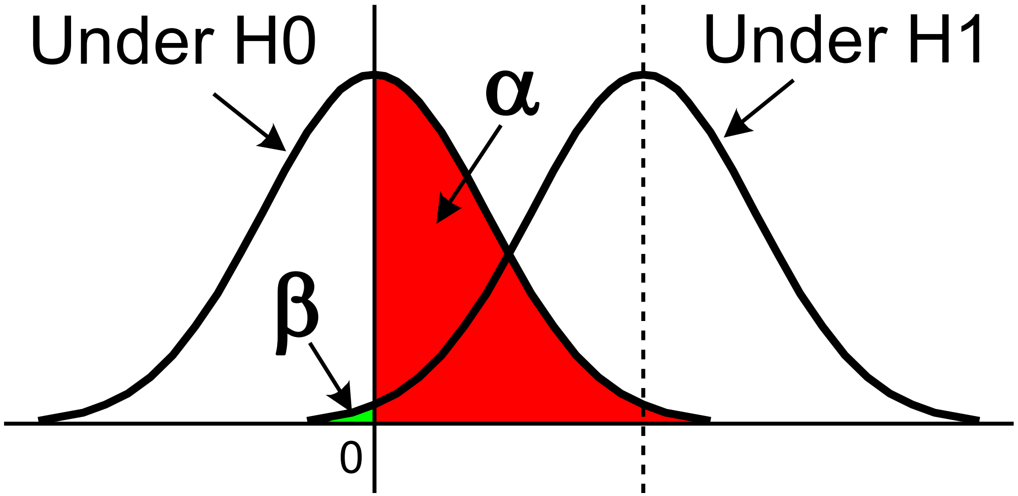
Two types of errors
Type I: not merging similar segments
Type II: merging different segments

α = Prob(Type I errors)
β = Prob(Type II errors)



Select the threshold to minimise α or β, but not both simultaneously

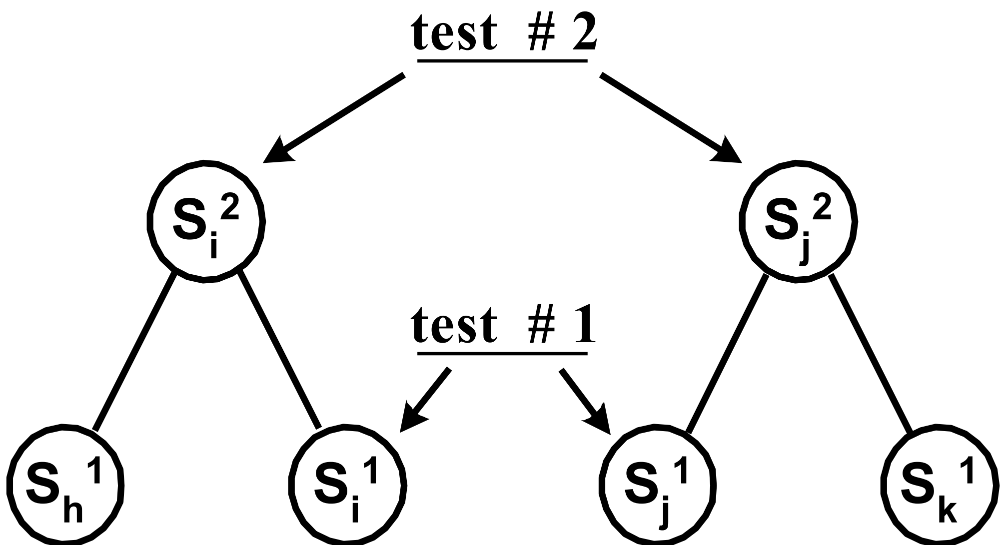
In hierarchical segmentation, type II errors (merging different segments) can not be corrected, while type I errors can be corrected later on.



The distribution of H1 and β are unknown.
Reduce β by increasing α.

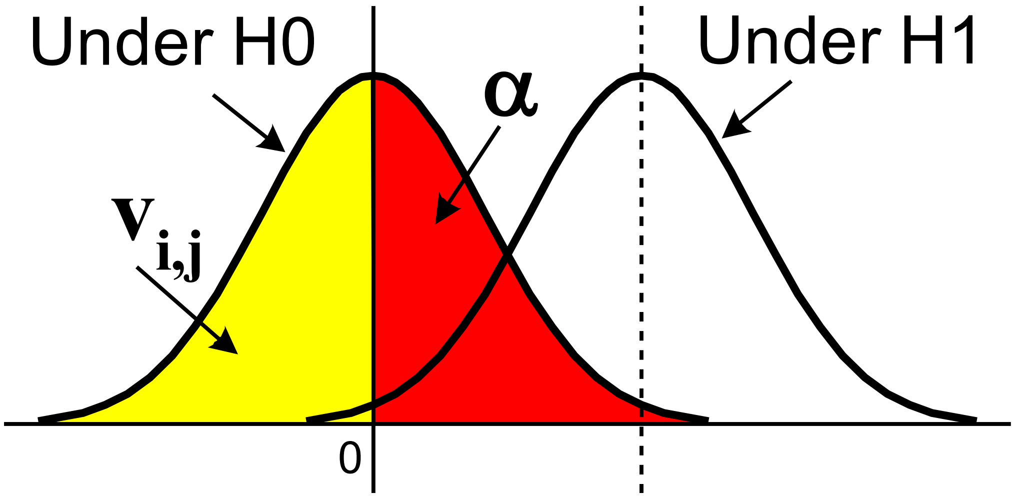
Sequential testing:
α will be reduced as segment sizes increase.

$\alpha_{1+2+\ldots} \leq$ **minimum(** $\alpha_1, \alpha_2, \ldots$ **)**
$\beta_{1+2+\ldots} \geq$ **maximum(** $\beta_1, \beta_2, \ldots$ **)**



Stepwise criterion

Find and merge the segment pair (i, j) that minimizes $V_{i,j}$ (= 1 - α).



$$V_{i,j} = \text{Prob}(d \le d_{i,j} ; H0) \quad (= 1 - \alpha).$$