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SEGMENTATION OF RANGE IMAGES BY PIECEWISE APPROXIMATION WITH SHAPE CONSTRAINTS

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RESUME

L'approximation fonctionnelle de l'image est une approche très utile pour la segmentation des images de profondeur (3D). Un algorithme de segmentation hiérarchique avec optimisation par étape est utilisé pour transformer un problème d'optimisation global en un problème d'optimisation séquentielle. Le critère d'étape correspond alors à l'accroissement de l'erreur d'approximation produit par la fusion de deux segments. Un modèle d'approximation par plan incliné est utilisé pour segmenter une image de profondeur formée de polyèdres, dont les résultats sont analysés. Des contraintes sur la longueur des contours des segments et sur leur forme sont ensuite ajoutées pour améliorer les résultats.

ABSTRACT

Piecewise functional approximation of picture is shown to be a useful tool for the segmentation of range (3D) image. A hierarchical step-wise optimization algorithm is employed to transform the global optimization problem into one of sequential optimization. The step-wise criterion then corresponds to the increase of the approximation error produced by the merge of two segments. The segmentation results of range image of polyhedra are shown with the utilization of a planar approximation model. Constraints on segment contour length and segment shape is then added to improve the results.

KEYWORDS: Three-dimensional vision, hierarchical segmentation, picture approximation, segment shape.

I - INTRODUCTION

Picture segmentation often constitutes the low-level processing stage of a picture analysis system. An image is thus segmented into regions that roughly correspond to surfaces or parts of objects of the scene. Constant gray level regions or more generally, planar regions are commonly used as "primitives". In robotics applications, the scene could be regarded as composed of polyhedra, cylinders, spheres, and other relatively simple curved objects. The identification of the objects can then be made from a range (3D) image where the "gray level", $f(i,j)$, corresponds now to the elevation of the object surface at position i,j . A segmentation process would thus be used to distinguish each of the surfaces, and a higher level process would combine these regions to identify the objects. Each segment could therefore be represented by a function that approximate the corresponding surface.

Haralick [5] presents a facet model based upon functional approximation. The picture is divided into overlapping cells, and the slope of an approximating plane is calculated for each cell. The slope value is used to decide whether an edge occurs between cells, otherwise the cells are merged into a same region.

Pavlidis [7], [8] presents the picture segmentation as a problem of piecewise functional approximation. Using approximation theory, he derives an algorithm for waveform approximation which locally optimizes an error criterion [7]. He also presents an algorithm where a picture is sliced into thin strips [8]. Each strip is segmented and approximated by polynomials. Then, adjacent segments with similar polynomial coefficients are merged into regions.

Horowitz and Pavlidis [6] propose a split-and-merge approach using a pyramid data structure. The data structure defines the way in which segments can be merged or split. A pyramid is a stack of regular picture blocks of decreasing sizes. The picture blocks (or segments) of one level are split into four regular sub-parts to form the next lower level. A pyramid can be regarded as a segment tree where each node corresponds to a

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block of $2^k \times 2^k$ pixels. A segment is considered as homogeneous if the segment approximation error is smaller than a predefined threshold. The algorithm consists of 1) merging the homogeneous segments, if the resulting segments are also homogeneous, or 2) splitting the segments that are not homogeneous into their four sub-parts.

The main difference between approximation approach and other segmentation approaches is that the former optimize a global criterion: the error norm. An important advantage is that the process is not misled by local unexpected pixel values. However, a severe disadvantage is the large amount of processing time and computer memory required.

Beaulieu and Goldberg [1], [2] propose a Hierarchical Step-Wise Optimization (HSWO) algorithm, which uses a hierarchical constraint to reduce the search space. It is shown that under this constraint the algorithm produce the best picture approximation. The algorithm is designed so as to reduce the computing time. Recalculations are avoided by 1) making explicit the information needed, and 2) updating the only values that are modified by a segment merger [3].

The Hierarchical Step-Wise Optimization algorithm and the picture approximation problem are presented in the next sections. In section IV, the planar approximation is examined and applied upon a range image. A local smoothness measure is used to guide the first segment merging where the approximation is under-determined. In section V, constraints upon contour length and segment shape are introduced to improve the segmentation results.

II - THE HIERARCHICAL STEP-WISE OPTIMIZATION ALGORITHM

A hierarchical segmentation algorithm based upon step-wise optimization is used in this paper [1], [2]. A segment similarity measure, $C(i,j)$, is defined as the step-wise criterion to optimize. At each iteration, the algorithm employs an optimization process to find the two most similar segments, which are then merged.

The Hierarchical Step-Wise Optimization (HSWO) algorithm can be defined as follows:

- i) Define an initial picture partition.
- ii) For each adjacent segment pair, (S_i, S_j) , calculate the step-wise criterion, $C(i,j)$; then find and merge the segments with the minimum criterion value.
- iii) Stop, if no more merges are needed; otherwise, go to ii).

Different segment similarity measures (step-wise criteria) can be employed, each one corresponding to different definitions of the picture segmentation task. The case of functional approximation is now considered.

III - PICTURE APPROXIMATION

Let $f_i(x,y)$ designate the pixel values for the segment S_i , ($f_i(x,y)=f(x,y)$ for $(x,y) \in S_i$). Piece-wise picture approximation, therefore, consists in approximating each segment by a polynomial function, $r_i(x,y)$. The approximation error for each segment is defined as the sum of the squared deviations:

$$H(S_i) = \sum_{(x,y) \in S_i} [f_i(x,y) - r_i(x,y)]^2 \quad (1)$$

The goal of picture approximation is then to find the partition, $\{S_i\}$, that minimizes the overall approximation error, $\sum H(S_i)$.

The segment similarity measure, thus, can be related to the increase of the approximation error produced by the merging of two segments, S_i and S_j :

$$C(i,j) = H(S_i \cup S_j) - H(S_i) - H(S_j) \quad (2)$$

The utilization of $C(i,j)$ in the HSWO algorithm ensures that each iteration does it best to minimize the overall approximation error. Segmentation results produced by planar approximation are now examined.

IV - PLANAR APPROXIMATION OF RANGE IMAGE

Figure 1 shows an image produced by a laser range finder based on a synchronized laser scanner [9]. It is composed of superposed polyhedra. A planar approximation model is used to segment the picture. The approximation function is :

$$r_i(x,y) = a_i + b_i(x) + c_i(y) \quad (3)$$

This function is employed in the calculation of $C(i,j)$, and the details of the implementation are given in [3]. The segmentation results are shown in Figure 2.

In the criterion calculation, it is assumed that each pixel is a square elementary area of constant value (horizontal surface) as shown in Figure 3. Therefore, the approximation error for constant value regions will have a tendency to be smaller than for inclined regions. This difference is usually a very small part of the segment approximation error. It is important only for the merging of segments with few pixels (1 or 2 pixels) where the approximation is under-determined. The formation of constant value regions is thus promoted during the first segment merges. While this can be advantageous for many picture segmentation problems (e.g. in remote sensing [3]), this produces unacceptable artifacts for range images. The formation of elongated constant value regions is promoted, as well as the merge of those strips even if they come from different plane surfaces. This explains the presence of the "L" shaped regions in Figure 2.

In order to cancel the advantage of constant regions, each pixel is regarded as a dimensionless point, the approximation error corresponds therefore to the distance between these points and

the approximating plane (see Figure 4). However, a new criterion must now be introduced to guide the first segment merging, the merge of 1 or 2 pixels segments. For each pixel, the following local homogeneity measure is defined:

$$\text{homo}(x,y) = \text{ABS}[(f(x+1,y)+f(x-1,y))/2 - f(x,y)] + \text{ABS}[(f(x,y+1)+f(x,y-1))/2 - f(x,y)]$$

$$\text{Mh}(S_i) = \sum_{(x,y) \in S} \text{homo}(x,y) / N_i \quad (4)$$

where $\text{homo}(x,y)$ can be regarded as a local measure of noise or as a measure of flatness. $\text{Mh}(S_i)$ is the mean value for segment S_i , and N_i is the size of segment S_i . The new step-wise criterion is therefore:

$$\text{Ch}(i,j) = C(i,j) + \text{Mh}(S_i \cup S_j) \quad (5)$$

This will promote the initial merging of pixels inside flat and noiseless regions. As segments become larger, the Mh part of the criterion will stay small, while $C(i,j)$ will increase largely. Mh is an average value that becomes more stable and constant over the picture as segments become larger. $C(i,j)$ measures the increase in the sum of the approximation error, and the increase becomes more important as segments become larger.

V - CONTOUR LENGTH AND SEGMENT SHAPE CONSTRAINTS

The step-wise optimization algorithm can employ different similarity measures, which correspond to different segment description models. Picture approximation is a simple and powerful model. However, more complex models could be required for segmentation tasks. Complex measures could be obtained from the combinations of simpler ones.

Zobrist and Thompson [10] point out that human vision employs many cues such as brightness, contour, color, texture and stereopsis to perform perceptual grouping. They stress the limitations of using only one cue at a time for computer grouping, and show the importance of studying mechanisms that combine many cues.

Constraints on the segment contour length and on segment shape is now added to the planar approximation model. Following an approach similar to Brice and Fennema [4], the first constraint will limit the increase of the segment contour length.

$$Cl(i,j) = \text{length}(S_i \cup S_j) / (\text{length}(S_i) + \text{length}(S_j)) \quad (6)$$

where $\text{length}(S_i)$ is the contour length of the segment S_i . Minimum and maximum values for the measure are 0.0 and 1.0, more common values are between 0.4 and 0.8. A low value indicates a large common boundary, and the merge of those segments should be promoted.

A segment shape measure is also used to promote the formation of compact regions instead of elongated ones. Let us define

$$\begin{aligned} mx &= \sum x / n \\ my &= \sum y / n \\ vx &= \sum (x-mx)^2 / n \\ vy &= \sum (y-my)^2 / n \end{aligned} \quad (7)$$

where the summation is over $(x,y) \in S_i \cup S_j$, and n is the size of $S_i \cup S_j$. The shape measure is therefore:

$$Cs(i,j) = \sqrt{vx / n} + \sqrt{vy / n} \quad (8)$$

Common values for the measure are between 0.5 and 2.0. A low value indicates a compact region, and a larger value corresponds to a longer region.

A new composite step-wise criterion is obtained in the following way:

$$C_{\text{new}}(i,j) = \text{Ch}(i,j) * (Cl(i,j) - a1) * (Cs(i,j) - a2) \quad (9)$$

where $a1$ and $a2$ are two parameters to control the importance of the contour length and segment shape constraints. Good results are obtained with a value of 0.2 for both parameters.

This new criterion is employed to segment the range image of Figure 1, and the results are shown in Figure 5. Each polyhedron surfaces are now correctly detected. Each segment of Figure 5-a can be associated easily with a distinct surface of the range image. The only exception is region #1. In Figure 5-b, more segments are allowed in order to reduce the approximation error. The new regions generally occur in corner areas or at step edges. At the intersection of 3 surfaces (corner) of Figure 5-a, a new plane is added in Figure 5-b in order to obtain a better approximation of the elevation transition. Regions #2, #3, #4 and #5 are such examples. The areas that correspond to almost jumps in the elevation values (step edges) could require the addition of many small planes in order to reduce the approximation error. This is illustrated by the small regions added at the bottom of the "L" shaped polyhedron in Figure 5-b.

VI - CONCLUSION

Picture approximation is an appropriate approach for the segmentation of range images. Beaulieu and Goldberg [1], [2] have shown the advantages of the Hierarchical Step-Wise Optimization algorithm for picture approximation problems. This algorithm is employed for the segmentation of a range image composed of superposed polyhedra. A planar approximation model is used and a local flatness measure is introduced to guide the first segment merges where the approximation is under-determined. Constraints upon contour length and segment shape is also employed to improve the results. The detection of polyhedron surfaces is correctly performed. Each obtained segment can be associated easily with a distinct surface of the range image.

We are now exploring approximation functions that would be more appropriate for the extraction and representation of cylinders and spheres in range images.

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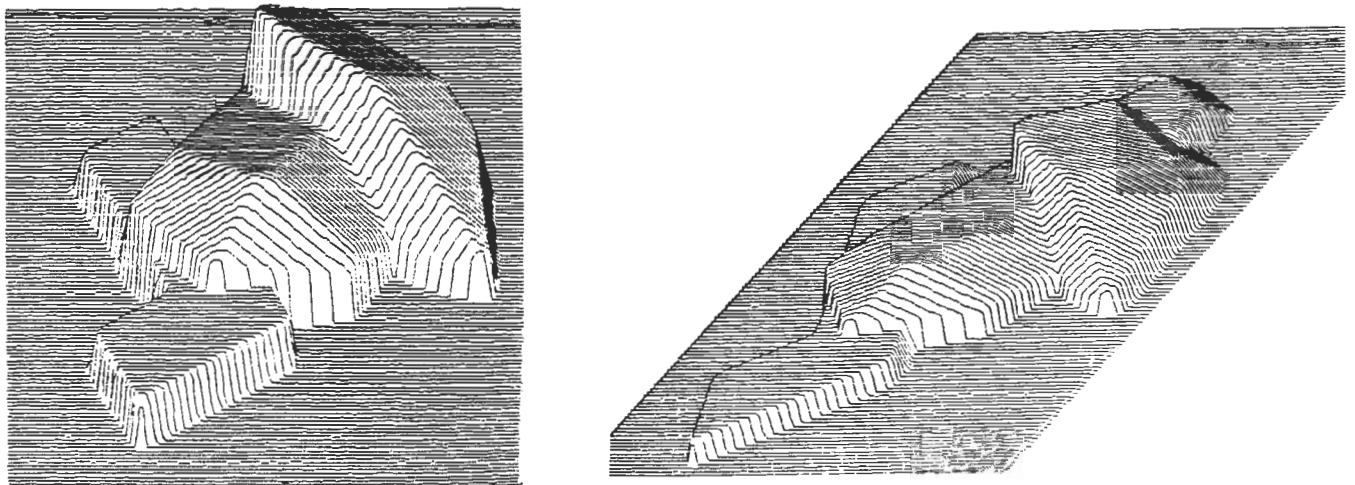
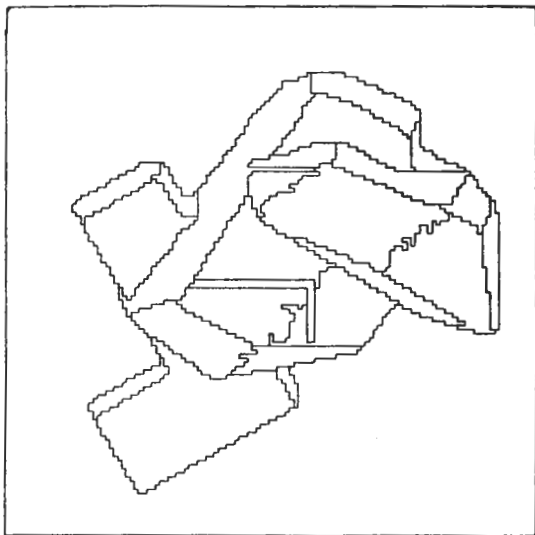
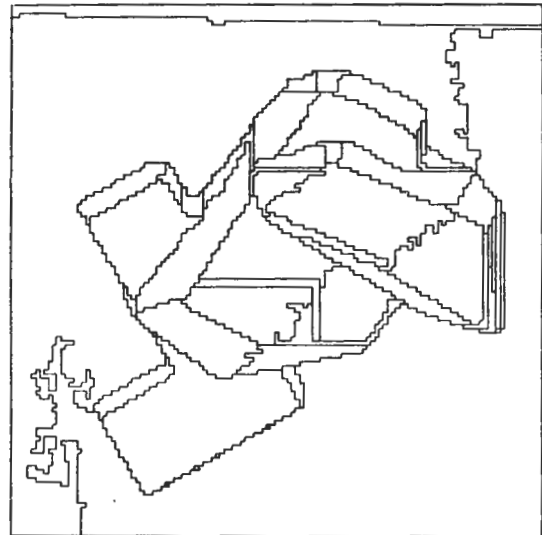


Figure 1: Range image of superposed polyhedra.



a) 25 segments



b) 50 segments

Figure 2: Planar approximation segmentation results where each pixel is a constant elementary area.

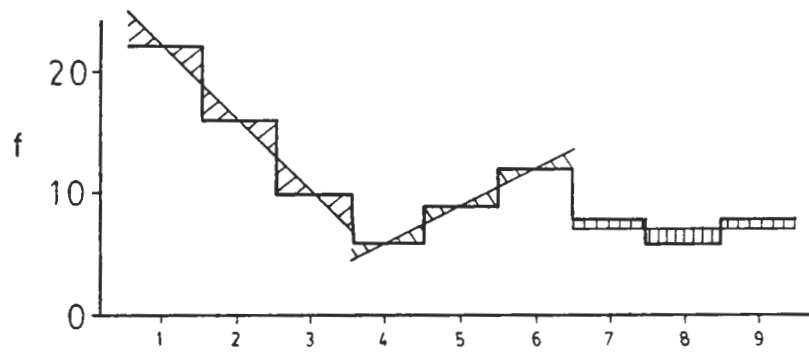


Figure 3: A one-dimensional example of planar approximation where each pixel is a constant elementary area.

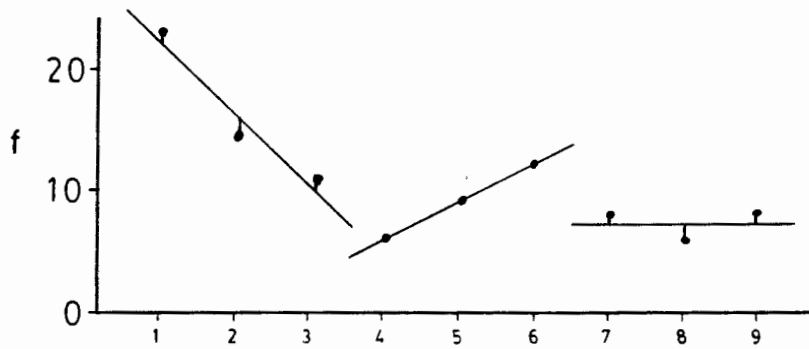
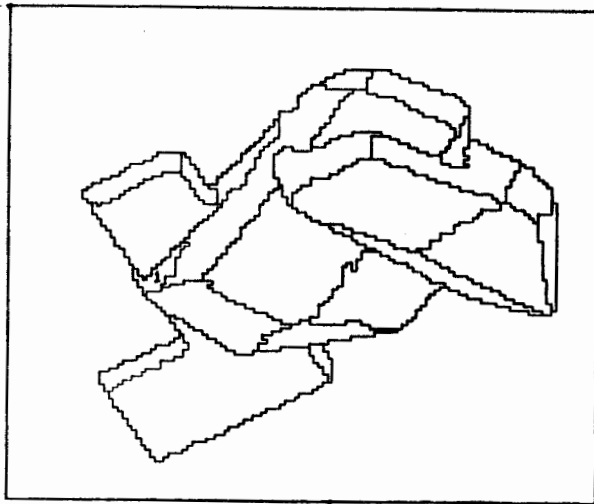
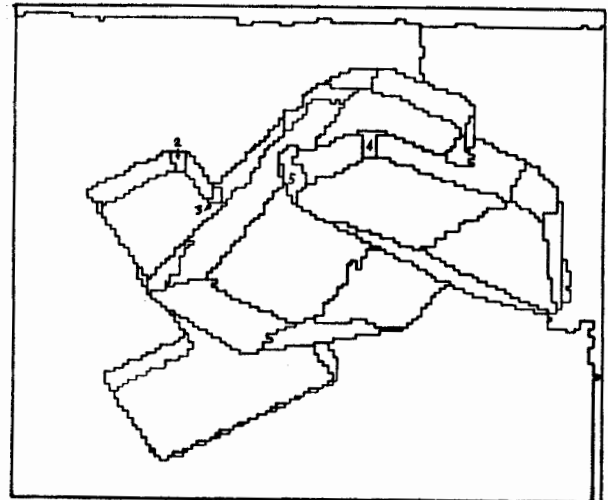


Figure 4: A one-dimensional example of planar approximation where each pixel is a dimensionless point.



a) 25 segments



b) 50 segments

Figure 5: Planar approximation segmentation results with constraints upon contour length and segment shape.